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Research Paper

Analytical solution and numerical simulation of vacuum consolidation by vertical drains beneath circular embankments

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A B S T R A C T

This paper presents an analytical solution and numerical simulation of vacuum consolidation beneath a circular loading area (e.g. circular oil tanks or silos). The discrete system of vertical drains is substituted by continuous concentric rings of equivalent drain walls. The effectiveness of the vacuum as distributed along the drain length and the well resistance of the drains are considered. A rigorous solution of radial drainage towards cylindrical drain walls is presented and compared to numerical FEM predictions. The model is then successfully adopted to analyse the vacuum consolidation of a circular embankment in the Ballina field testing facility in Australia.

1. Introduction

Very soft clays are widespread in many coastal regions of Australia and other parts of the world. These soft clay deposits normally show unfavourable soil properties such as low bearing capacity and high compressibility. For such soils, appropriate ground improvement techniques are usually employed to minimize the post-construction settlements and lateral displacements which may threaten the stability of infrastructure built on them [16].

Application of prefabricated vertical drains (PVDs) is one of the most widely used ground improvement techniques for improving the mechanical properties of soft clay deposits. The PVDs expedite the progress of soil consolidation by shortening the length of the drainage path and enhancing radial drainage. PVDs are artificially created vertical drainage boundaries which accelerate lateral (radial) flow from the surrounding soil, thereby increasing the rate of consolidation significantly. The consolidation of the soil results in higher shear strength and helps to reduce the post-construction settlement of the super-structure [13]. However, preloading with PVDs can be relatively slow, as a staged construction is generally required to prevent instability. In such cases, the application of vacuum pressure would significantly reduce the consolidation time (e.g. [9,23]).

The behaviour of vertical drains was first solved analytically by Barron [1] and Richart [33] based on a unit cell concept. The unit cell represents a single drain surrounded by a soil annulus under axisymmetric conditions (three-dimensional, 3D). Hird et al. [12] extended the unit cell concept to the plane strain condition (two-dimensional, 2D). The unit cell concept is accurate when applied at the embankment centreline, where the lateral displacements are negligible. In practice, the subsoil is usually not uniform, and the process of consolidation is not always a one-dimensional problem [15]. Numerical modelling of multidrain systems in plane strain was further improved by Indraratna and Redana [20], who introduced a mathematical technique to convert axisymmetric properties to equivalent 2D plane strain condition and also considered the smear effect caused by mandrel intrusion.

The vacuum preloading (VP) method was initially introduced in Sweden by Kjellman [26] for cardboard wick drains. The 2D analysis is also applicable for VP in conjunction with vertical drains (e.g. [10]). Mohamedelhassan and Shang [28] proposed an analytical solution for radial consolidation with vacuum application. Indraratna et al. [23] extended the unit cell radial consolidation theory for vacuum application, based on the equivalent plane strain condition and considering the potential vacuum loss along the drain length. It can be noted that most of the previous studies have been devoted to modelling the multidrain systems corresponding to an embankment strip loading in 2D plane strain. So far, no study has been conducted to model vacuum consolidation via PVDs beneath a circular loaded area, where the system

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conforms to an axisymmetric problem. Ground improvement, however, may be needed for most heavy circular structures, such as oil or water storage tanks, silos, and heavily loaded roundabouts in commercial areas. Indraratna et al. [14] first introduced the concept of concentric rings of equivalent drain walls for simulating circular embankments with PVDs, but this work was limited to conventional preloading with vertical drains and no vacuum pressure.

The main objective of this paper is to introduce a comprehensive analytical solution for vacuum preloading in conjunction with vertical drains beneath a circular foundation. The theory of ring walls is extended to accommodate vacuum preloading, considering the possible loss of vacuum with depth and the effect of well resistance on consolidation. The finite element model (FEM) in PLAXIS [5], incorporating the authors' solution, is then validated for a single ring situation. The FEM code PLAXIS is then used to analyse the performance of a full-scale test embankment to be constructed near the Pacific Highway in Ballina, NSW, Australia. The effect of vacuum pressure termination on the consolidation response and post-construction settlements are also analysed and a novel approach to determine the optimum time for terminating the vacuum application is presented and examined for the design of the Ballina circular embankment.

2. Mathematical formulation

Vertical drains are generally installed either in equilateral triangular or square patterns. It is noted that the square pattern can be more easily controlled in the field, although the triangular pattern may give more uniform settlement [34]. The vacuum consolidation of soil around a single vertical drain can be readily analysed as a unit cell [23]. However, to analyse a multidrain system under an axisymmetric condition, the equivalent soil parameters that give the same time-settlement response in the field must be determined. Indraratna et al. [14] proposed a transformation scheme for axisymmetric problems in which each drain is assumed as a part of the concentric cylindrical drain wall with an increasing perimeter with the radial distance from the centreline, as shown in Fig. 1. In this section, the ring wall theory is extended to incorporate the concept of vacuum consolidation for the analysis of circular loading, such as silos or oil and water storage tanks.

2.1. Assumptions

The main assumptions made in developing the analytical solutions are summarized below:

- Equal strain assumption and small strain theory are valid. The flow in the soil mass is assumed to be laminar and Darcy's law is adopted.
- Only vertical strains are allowed, i.e. at the centreline of a relatively large loading area, volume change is due to settlement only, and lateral displacements are negligible.
- The soil is fully saturated and homogeneous, and the permeability of the soil is assumed to be constant during consolidation.
- Well resistance is taken into account. It is assumed that well resistance is constant during consolidation.
- Each set of vertical drains located at the same radial distance from the line of axisymmetry is modelled as a continuous cylindrical drain wall of radius $r_i = i \cdot S$, where S is the spacing of the drains and i is the number of that set, as shown in Fig. 1(c).
- It is assumed that the cylindrical drain wall has a negligible thickness.
- Each cylinder is assumed to be impermeable with respect to the outer and inner boundaries, and has an internal horizontal (radial) flow.
- Smear effects around the drain walls are not incorporated directly in the equations. It is assumed that the smear effect due to drain installation can be taken into account in by calculating the reduced lateral permeability of the soil [18].
- The vacuum pressure distribution along the drain boundary is considered to vary linearly from $-p_0$ at the top of the drain to $-k_1 p_0$ at the bottom of the drain. The vacuum pressure is assumed to be evenly distributed in the horizontal direction between the adjacent drains.

2.2. Analytical solution

Considering the inner hollow cylindrical soil wall, the flow rate in the radial direction from the inner impermeable boundary to the hollow cylindrical drain wall is expressed by Darcy's law, i.e. in zone (1) of Fig. 2(a):

$$\frac{\partial Q}{\partial t} = \frac{k_h}{\gamma_w} \frac{\partial u}{\partial r} A \tag{1}$$

where Q is the flow in the soil mass, u is the excess pore pressure due to preloading, and A is the cross-sectional area of the flow at distance r and can be expressed as $(2\pi r dz)$ where dz is the height of an arbitrary thin layer of the soil as shown in Fig. 2(b). γ_w is the unit weight of water, t is time and k_h is the horizontal permeability coefficient of soil.

The rate of volume change in the vertical direction of the soil mass can be expressed by:

$$\frac{\partial V}{\partial t} = \frac{\partial \varepsilon_v}{\partial t} \pi \left[\left(r_i - \frac{S}{2} \right)^2 - r^2 \right] dz \tag{2}$$

where V is the volume of the soil mass and ε_v is the volumetric strain.

The rate of radial flow is assumed to be equal to the rate of volume change of the soil mass in the vertical direction, therefore, by rearranging Eqs. (1) and (2), the gradient of excess pore pressure can be derived as:

$$\frac{\partial u_1}{\partial r} = \frac{\gamma_w}{2k_h} \frac{\partial \varepsilon_v}{\partial t} \frac{1}{r} \left[\left(r_i - \frac{S}{2} \right)^2 - r^2 \right] \tag{3}$$

where the index 1 refers to zone 1 in Fig. 2(a).

Similarly, the gradient of excess pore pressure in zone (2) of Fig. 2(a) is determined by:

$$\frac{\partial u_2}{\partial r} = \frac{\gamma_w}{2k_h} \frac{\partial \varepsilon_v}{\partial t} \frac{1}{r} \left[\left(r_i + \frac{S}{2} \right)^2 - r^2 \right] \tag{4}$$

Considering a horizontal cross-sectional slice with thickness dz in Fig. 2(b), the change of flow in the z direction of the drain from the entrance to the exit of the slice, dQ_z, is then expressed by:

$$dQ_z = \frac{q_w}{\gamma_w} \frac{\partial^2 u}{\partial z^2} dzdt \tag{5}$$

where the flow term q_w represents well resistance.

The total change in flow from the inner impermeable boundary to the exit face of the slice (dQ_1) is given by:

$$dQ_1 = \frac{2\pi r_i k_h}{\gamma_w} \frac{\partial u_1}{\partial r} dzdt \tag{6}$$

Similarly, the total change in flow from the outer impermeable boundary to the exit face of the slice (dQ_2) is equal to:

$$dQ_2 = \frac{2\pi r_i k_h}{\gamma_w} \frac{\partial u_2}{\partial r} dzdt \tag{7}$$