Computers and Geotechnics 80 (2016) 178–189

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

A novel finite element two-step solution scheme for fully coupled hydro-mechanical processes in poroelastic media

Gen Li [a,b,c,*], Chun-An Tang [a,b,c], Lian-Chong Li [a,c], Hong Li [a,c]

[a] State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian 116024, China
[b] State Key Laboratory for Geomechanics and Deep Underground Engineering, China University of Mining and Technology, Beijing 100083, China
[c] Ocean Engineering Joint Research Center of DUT-UWA, Dalian 116024, China

ARTICLE INFO

Article history:
Received 23 November 2015
Received in revised form 27 April 2016
Accepted 24 June 2016

Keywords:
Coupled hydro-mechanical analysis
Fully explicit
Precise integration method
Pressure formulation
Poroelastic

ABSTRACT

A novel two-step solution scheme (TSSS) is described for fully coupled hydro-mechanical (FCHM) analysis of saturated poroelastic media. The TSSS is based on the *pressure formulation* in two-step. In step one, the pressure field is obtained directly by solving the sub-problems with a reduced scale of displacement variables. This process is fully decoupled. In step two, the displacement field is calculated by staggered iteration of pressure variables. The finite element method (FEM) is used for discretization of the FCHM differential equations in the space domain. The precise time step integration method is performed for the time derivatives. The stability and convergence of the TSSS are proved using a matrix-based spectral analysis in the time domain. It is demonstrated that the TSSS is unconditionally stable, fully explicit and highly precise. The algorithmic error estimation results indicate that the numerical performance in the time domain can match the computer precision. Theoretically, the algorithmic error is caused only by mesh discretization. The stability and accuracy of the TSSS are verified and calibrated by numerical examples. By comparing with the analytical or reference solutions, it is shown that the TSSS result is highly precise, and it is remarkably better than the standard FEM in terms of precision. In addition, the numerical results are stable and independent of the time step size. The numerical experiments also demonstrate the stability, convergence and precision for the *pressure formulation* of TSSS. The proposed scheme has great potential in engineering applications for long timescale problems, especially the problems focusing on the evolution of pressure field.

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1. Introduction

The interaction between the pore fluid and solid skeleton of porous geomaterials is a fully coupled fluid-flow and solid-deformation problem (such as rocks, and soils), which has been studied in many engineering fields, such as reservoir engineering, hydraulic and hydropower engineering, geotechnical and environmental engineering [1,2]. The theoretical basis can be dated back to the mid-20th century, and the mathematical framework can accurately describe the fully coupled hydro-mechanical (FCHM) process of saturated poroelastic media [2–4]. However, due to the complex physical interactions and complicated boundary conditions, the analytical solutions of FCHM problems are available in only a few situations. The extensive use of computers and the concomitant development of numerical techniques have made precise analysis possible for practical problems and shifted attention from problem solving to modeling observed phenomena. The eventual aim is to develop the capability for making high-efficiency and precise simulators [1,5,6]. In order to achieve this goal, numerical techniques have developed and extended towards different directions, such as the numerical approach pertinent to differential equations [1,7–13], and the numerical stabilized formulation [14–19] and solution strategies.

After spatial discretization by a numerical approach, the ordinary differential equations are transformed into monolithic equations. The monolithic equation is commonly discretized in time. Then, three solution strategies can be adopted in the discretization system, as direct evaluation, partitioned schemes and two single-field analysis. Direct evaluation [1,20], i.e. the fully coupled **u**-*p* equation system directly solved simultaneously for the solid displacements **u** and fluid pressures *p* at every time step. In such case, numerical efficiency becomes the primary concern. The overall efficiency depends to a great extent on the algorithm used for solution of the algebraic equation system resulted from the

* Corresponding author at: Dalian University of Technology, No. 2 Linggong Road, Ganjingzi District, Dalian City, Liaoning Province, China.
E-mail address: ligen8@dlut.edu.cn (G. Li).

http://dx.doi.org/10.1016/j.compgeo.2016.06.015
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discretization process. However, with scale of the fully coupled **u**-*p* equation system increasing, the computational cost and memory requirements for the scheme has rised up dramatically, especially for long timescale problems. Hence, the sequential solution methods have been proposed later. Partitioned schemes, including simultaneous and staggered solutions [21–25]. In the partitioned schemes, an appropriate partitioning of the matrices is performed on the algebraic system allowing the equations to be solved sequentially. Either the flow or mechanical problem is solved first, with the other field variables frozen. Then the other problem is solved using the intermediate solution information. One may iterate this sequential procedure at each time step until convergence [21–25]. During this process, the coupled problem is partitioned and the sub-problems are solved sequentially. The partitioning allows for the use of existing robust simulators for the sub-problems, producing smaller systems of equations to be solved than the fully coupled methods. However, the stability and convergence of the sequential schemes can be quite different when addressing various problems [24,25], which leads to difficulties in practical applications. Therefore the correctness and precision of these sequential schemes should be carefully assessed each time [1]. The fully coupled "true" solution is always the target of these sequential schemes. The fully coupled method is internally consistent and rigorous because the fluid flow and the solid deformation are solved simultaneously on the same discretized grid [26].

In many practical problems, only the **u**/*p* field or the physical field for a particular moment is concerned. For instance, the long-term surface subsidence connected with the change of hydraulic equilibrium in systems comprising aquitards and/or hydrocarbon reservoirs may be the focus, when there is extensive groundwater withdrawal and/or oil and gas pumpage for a long time; on the contrary, the distribution and evolution of the *p* field is more concerned during hydraulic fracturing in oilfield, rather than the distribution of the **u** field. In this case, either the **u** or *p* field variables can be eliminated from the time-discretized equation. Smith [27] proposed the so-called *displacement formulation*, which yields directly the **u** field by eliminating *p*. Alternatively, if **u** is eliminated from the monolithic equation and a *pressure formulation* is obtained after discretization in the time domain, which was done by Krause [28]. Either the *displacement* or *pressure formulation* can be used, i.e. the coupled two-step procedure (or the two single-field analysis), if both fields are concerned. It should be emphasized that the two-step procedure gives the fully coupled solutions, which is consistent with the results of direct evaluation. The elimination process can be considered as the decoupling process for the **u** or *p* field variables. The sub-problems with reduced scale can then be solved.

However, it is worth noting that no matter which numerical approach and solution strategy is adopted, the finite difference (FD) operator for the time derivative is always needed. According to the generalized trapezoidal θ-method, the variables and their temporal derivative in the time interval can be a two-point FD approximation [1,20], given that θ is an integration parameter varying from 1.0 (fully implicit) to 0 (fully explicit). The value of θ has an important influence on the stability. The approximation is unconditionally stable when $\theta \geqslant 0.5$, but for any value of $\theta \neq 1$, the numerical solution can exhibit a spurious rippling effect [29]. Thus the fully implicit ($\theta = 1$) scheme is commonly used. However, the computation precision and computational cost always exist in the FD-based schemes. The accuracy of the results is sensitive to the time step size. A large time step size would lead to large errors, but less time-consuming. In order to optimize this problem, to estimate the local truncation error and to control the time step size, the adaptive time-stepping methods and three-point discretization scheme have been developed [21,30–34]. Nevertheless, these FD based schemes are either first- or second-order accurate approximations, and therefore their accuracy is relatively low. When $\theta = 0$, even though the fully explicit algorithm is simple, it is stable for only an extremely small time step size, and perhaps the step size is so small that a real time of interest would require an excessive number of steps. Therefore, the fully explicit scheme is not of practical value. However, the explicit FCHM simulators are still attractive to users owing to their advantages in that there is no need to solve the linear equations of the global stiffness matrix (in contrast to the more commonly used implicit methods) and also no need to check the convergence of the solution. Thus, the mathematical operation and structure are simpler. The widely used FLAC software is an explicit FD program for geotechnical mechanics computation [7]. However, its algorithm is still conditionally stable, which requires strict control of the time step size. Regrettably, little attention is devoted to the high precise and unconditionally stable explicit FE modeling by which the numerical schemes are suitable for the long time scale FCHM problems. There is no doubt that the development of a time domain FEM simulator for the FCHM analysis, which is explicit, insensitive to the time step size and can be easily implemented, is appealing and of great practical value.

This paper presents a novel two-step solution scheme (TSSS) based on *pressure formulation* for the FCHM FE modeling of saturated poroelastic media. The proposed TSSS is unconditionally stable, highly precise and fully explicit, in which the precise integration method is employed for time derivatives. The initial numerical integration efforts towards the parabolic partial differential systems were presented in [35]. Section 2 describes the mathematical formulation of the FCHM model of saturated poroelastic media. In Section 3, a FE implementation of the TSSS is presented, and the stability and convergence of the TSSS are proved. The accuracy and stability of the numerical scheme are verified and calibrated in Section 4 by applications in typical geotechnical problems. The conclusions and remarks are presented in Section 5.

2. Coupled hydro-mechanical model of saturated poroelasticity

The physical model is based on poroelasticity theories [1,2], and assumptions are made on the fluid-saturated porous medium, isotropic geomaterials, isothermal single-phase flow of a slightly compressible fluid (water in this study), and a slow and small deformation process.

2.1. Differential equations

The governing differential equations for coupled flow and poroelastic media are based on the linear momentum balance and mass balance. Under the quasi-static assumption for solid-phase displacement, the equilibrium equation for mechanical deformation of the hydro-mechanical system can be written as:

$$\nabla \cdot \boldsymbol{\sigma} + \rho_b \mathbf{g} = \mathbf{0} \tag{1}$$

According to Terzaghi's effective stress principle that links the solid grain stress to the fluid pore pressure

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - \alpha p \mathbf{I} \tag{2}$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}'$ are the total and effective Cauchy stress tensor (tension in the solid phase is positive), respectively; $\alpha = 1 - \frac{K_{dr}}{K_s}$ is the Biot's coefficient, K_s is the bulk modulus of the solid grain, and K_{dr} is the drained bulk modulus, p is the (excess) pore pressure (compression in the fluid phase is positive), $\mathbf{I}$ is the second-order unit tensor, ρ_b is the buoyant density of the saturated porous medium, $\mathbf{g}$ is the vector of gravity accelerations. The constitutive law of the solid phase is

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