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Research Paper

A stable Maximum-Entropy Meshless method for analysis of porous media

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ABSTRACT

Consolidation analysis of saturated porous media demands the coupling of solid displacements with the pore fluid pressure via the equilibrium and the continuity of mass. In this paper, a stable numerical procedure is presented for coupled analysis of consolidation problems in geotechnical engineering. The numerical framework is based on the Element-Free Galerkin method and the principle of Maximum Entropy. Identical shape functions are employed for approximating the displacement field as well as the pore fluid pressure field. The proposed method is used for analysing several consolidation problems assuming elastic and elastoplastic soil behaviour. The numerical results indicate that the proposed Maximum-Entropy Meshless method based on the maximum entropy shape functions is able to provide stable and robust solutions for consolidation problems in porous media.

1. Introduction

Meshless methods have significantly progressed during the past few decades. In these methods a set of nodes is used to discretise the domain of a problem and, hence, unlike the finite element (FE) method, connectivity between elements and nodal points does not exist. This concept implies that meshless methods are more appropriate than the FE method for tackling geotechnical problems including extremely large deformations, discontinuities due to crack propagation and separation of materials, moving boundary conditions, and strain localisation. It is noteworthy that in geotechnical applications, the continuum usually includes a solid phase (soil particles) and a fluid phase (water), demanding a robust and stable computational method to model two-phase response. Among others, the Element-Free Galerkin (EFG) method, developed by Belytschko et al. [2], has attracted significant attention for solving geotechnical problems, due to its robustness, relatively high accuracy, and superior convergence, e.g. see [13,21,29,31]. In the original EFG method, Moving Least Square (MLS) shape functions were employed to approximate unknown field variables [1]. MLS shape functions do not satisfy the Kronecker delta property, creating technical difficulties and extra computational challenge for imposing the essential boundary conditions. Recently, the concept of maximum entropy (max-ent) shape functions was introduced to the EFG method, which facilitates application of boundary conditions [1,22,24]. This technique is usually referred to as the Maximum-Entropy Meshless (MEM) method. Due to their robustness, the max-ent shape functions within the framework of the EFG method have been widely used in engineering problems. Examples include incompressible elasticity [14], nonlinear analysis of reinforced concrete structures [17], and analysis of thin shells [8]. More recently, these shape functions have been employed for adaptive meshless analysis [28] as well as combined FE-MEM methods of analysis of nonlinear solid problems [29].

Consolidation analysis of porous media by meshless methods has attracted significant attention. Among others, the consolidation phenomenon has been investigated using the EFG method [9,12,13,18,21], and the Radial Point Interior (RPI) method [6,5,32,33]. In such analyses the unknown variables include displacements, $\mathbf{u}$, as well as the pore fluid pressures, $\mathbf{p}$. The accurate prediction of pore water pressure has faced difficulties, for which stabilisation techniques have been proposed. The instabilities in a coupled $\mathbf{u}$-$\mathbf{p}$ analysis are mainly due to (a) the time integration scheme, (b) the inconsistency between the order of shape functions used for interpolating displacements and those employed for approximating pore water pressures. Within the framework of the RPI method, Khoshghalb and Khalili [6] developed a three-point time discretisation technique which is second-order accurate and avoids the spurious ripple effects observed in a two-point integration scheme. Shibata and Mukarami [21] observed instability in consolidation analysis by the EFG method and introduced a

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stabilising term into the weak form of governing equations. The main advantage of this technique is that the interpolation fields in the pore pressure field do not reduce the accuracy of numerical results. Nonetheless, the method demands extra computational challenge [21]. Regardless of the numerical method, another source of instability in analysis of consolidation problems is often due to the inconsistency between the degrees of displacement field and the pore water pressure field. Oliaei et al. [13] demonstrated that the same order of MLS shape functions for displacement and pore water pressure provides stability as well as accuracy of numerical results provided an implicit time-integration scheme is employed.

In this study, the MEM method is further extended to allow fully coupled analysis of fully saturated porous media. It is demonstrated that the proposed method is intrinsically stable (no stabilisation technique is employed), does not require special treatment of the essential boundary conditions, and can successfully tackle elasto-plastic consolidation problems in fully saturated porous media. This is mainly due to the fact that the max-ent shape functions tend to be much less sensitive to the discretisation of the domain as well as the diversity in values of the interpolation field [14].

The outline of the paper is as follows. The equations governing the consolidation of a porous continuum are presented within the framework of the MEM method in Section 2. Then, an implicit scheme based on the Backward Euler method for integrating the global equations is briefly discussed in Section 3. The maximum entropy principle and the corresponding shape functions are introduced in Section 4. In Section 5, several numerical examples are presented to validate the formulation and to demonstrate the stability of the proposed method. The key outcomes of this study and the conclusions drawn are summarised in Section 6.

2. Governing equations

In this section the equations governing the nonlinear behaviour of a two-phase porous medium, assuming small deformations, are presented. In geotechnical problems, deformations are usually coupled with the dissipation of excess pore fluid pressure. Numerical analysis of such problems requires the coupling of the equilibrium with the continuity equation through the principle of effective stresses and Darcy's law. Consider the continuum shown in Fig. 1, where the problem domain Ω is bounded by domain boundary Γ. The equilibrium is satisfied provided that

$$\sigma_{ij,j} + b_i = 0 \tag{1}$$

where σ denotes the total Cauchy stress vector, b is the body force vector, and a comma in the subscript represents a partial derivative with respect to the indicated variable. Denoting v_s as the velocity of soil particles and $\tilde{v}$ as the superficial velocity of the fluid relative to the soil skeleton, and assuming that the soil solids and the pore water are much less compressible than the soil skeleton, the conservation of mass can be expressed by

$$\frac{\partial v_{si}}{\partial x_i} + \frac{\partial \tilde{v}_i}{\partial x_i} = 0 \tag{2}$$

The boundary conditions in Eqs. (1) and (2) are

$$\begin{aligned} &\sigma_{ij} \cdot \bar{n}_j = T_i && \forall\, x \in \Gamma_T \\ &u_i = \bar{u}_i && \forall\, x \in \Gamma_u \\ &p_i = \bar{p}_i && \forall\, x \in \Gamma_p \\ &q = n_s \tilde{v}_{\cdot i} \bar{n}_i && \forall\, x \in \Gamma_q \end{aligned} \tag{3}$$

where $\bar{n}$ is the vector of unit outward normal at a point x on Γ, T represents the prescribed traction on the traction boundary Γ_T, u is the displacement vector, $\bar{u}$ denotes the prescribed displacement on the displacement boundary Γ_u, p is the pore water pressure vector, $\bar{p}$ represents the prescribed pore water pressure on the pore water pressure boundary Γ_p, q is the prescribed flow on boundary Γ_q, and n_s is the porosity of soil.

In the original EFG method, the Moving Least Square (MLS) shape functions were employed to approximate unknown field variables [1]. The MLS shape functions do not satisfy the Kronecker delta property, creating technical difficulties and extra computational challenge for imposing the essential boundary conditions. Maximum entropy (max-ent) shape functions [22] provide an alternative way to interpolate the unknown variables while satisfying the Kronecker delta property. Therefore, the weak form of Eq. (1) can be written as

$$\int_{\Omega} \sigma_{ij} \delta u_{i,j} d\Omega + \int_{\Omega} b_i \delta u_i d\Omega + \int_{\Gamma_T} T_i \delta u_i d\Gamma_T = 0 \tag{4}$$

where δu_i denotes the vector of virtual displacements. The principle of effective stress relates the total Cauchy stress to the effective Cauchy stress, σ', and the pore water pressure, p,

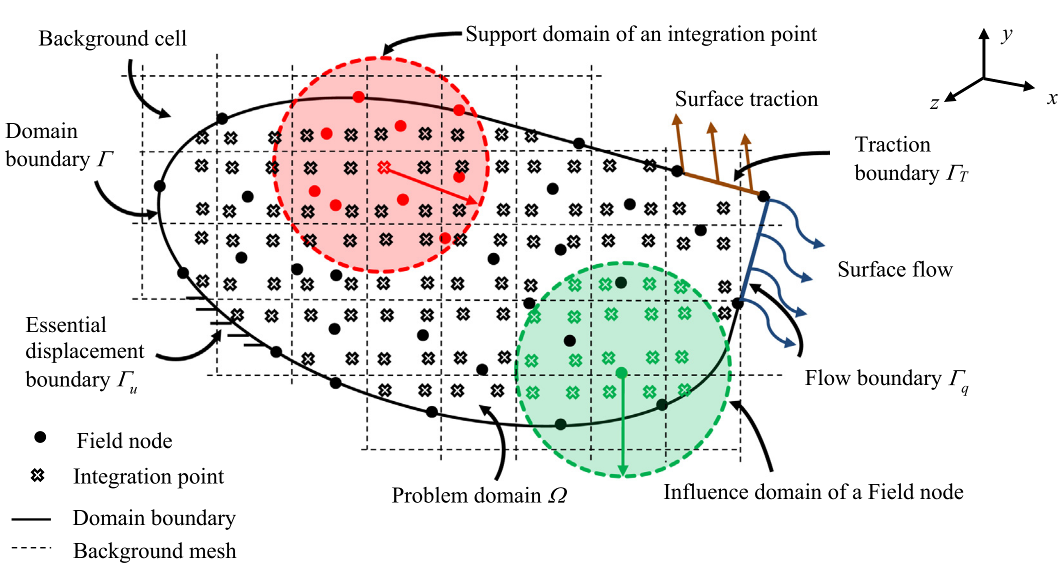

Fig. 1. Problem discretisation by the MEM method.