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Bingham fluid simulation with the incompressible lattice Boltzmann model

G.H. Tang*, S.B. Wang, P.X. Ye, W.Q. Tao

State Key Lab of Multiphase Flow, School of Energy and Power Engineering, Xi'an Jiaotong University, 28, Xianning West Road, Xi'an 710049, China

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ABSTRACT

The Bingham fluid flow is numerically studied using the lattice Boltzmann method by incorporating the Papanastasiou exponential modification approach. The He–Luo incompressible lattice Boltzmann model is employed to avoid numerical instability usually encountered in non-Newtonian fluid simulations due to a strong non-linear relationship between the shear rate tensor and the rate-of-strain tensor. First, the value of the regularization parameter in Bingham fluid mimicking is analyzed and a method to determine the value is proposed. Then, the model is validated by pressure-driven planar channel flow and planar sudden expansion flow. The velocity profiles for the pressure-driven planar channel flow are in good agreement with analytical solutions. The calculated reattachment lengths for a 2:1 planar sudden expansion flow also agree well with the available data. Finally, the Bingham flow over a cavity is studied, and the streamlines and yielded/unyielded regions are discussed.

1. Introduction

The studies of non-Newtonian fluids and the flow behaviors are of high interest to a broad range of disciplines in both science and technology, including hydrology, geophysics, materials, food, and biology. A plastic non-Newtonian fluid such as slurry, paste, paint, and margarine, only flows above a certain level of stress, called yield stress, while it exhibits little or no deformation below this yield stress. These materials are usually called Bingham plastics or Bingham fluids [1]. In many cases of interest, due to the limitation of analytical solutions it is highly desirable to find efficient numerical methods for such non-Newtonian flows under complex rheology properties and complicated bounded geometries. Though significant progress has been made in developing numerical approaches for such viscoplastic flows in various geometries, most of these schemes are based on traditional finite difference or finite element discretization in which a set of appropriate partial differential equations are discretized and solved [2–7].

The recently developed lattice Boltzmann method (LBM), due to its inherent vantages like simple implementation, high parallelizability and great convenience of handling complicated geometries and boundary conditions, has been successfully developed to study complex transport phenomena and model complex physics [8,9] which are usually hardly accessible to traditional macroscopic approaches. The kinetic essence of the LBM makes it also capable of calculating the local components of the stress tensor directly. The lattice Boltzmann models have been set up for non-Newtonian flow systems in the literature recently [10–20] by adjusting the relaxation time (and hence the viscosity) to the local shear rate. Thereinto, Wang and Ho [17] proposed a lattice Boltzmann model particularly for Bingham fluid by incorporating the effect of local shear rate into the lattice equilibrium distribution function, and a planar sudden expansion flow was examined. Vikhansky [18] proposed a novel and efficient version of the LBM for non-Newtonian flow simulation and the collisions are treated implicitly, i.e., the collision term is chosen such that the stress and strain rate tensors satisfy the constitutive equation after the collision. The method does not need any regularization and the Bingham flow was examined in his work. However, some velocity derivatives are introduced into the density equilibrium distribution function in Ref. [17], while in Ref. [18], a non-linear equation linking stress intensity with shear rate has to be solved at each node and each time step additionally. In this wok, still basing on the traditional lattice Boltzmann framework, we employ a fairly simple lattice Boltzmann scheme, the He–Luo incompressible model [21], with incorporation of the popular Papanastasiou exponential modification approach [22], to simulate the Bingham fluid flow.

2. Numerical methods

2.1. The Papanastasiou approach for Bingham fluid

In order to model the stress-deformation behavior of Bingham fluids, the ideal Bingham constitutive equations have been pro-

* Corresponding author.
E-mail address: ghtang@mail.xjtu.edu.cn (G.H. Tang).

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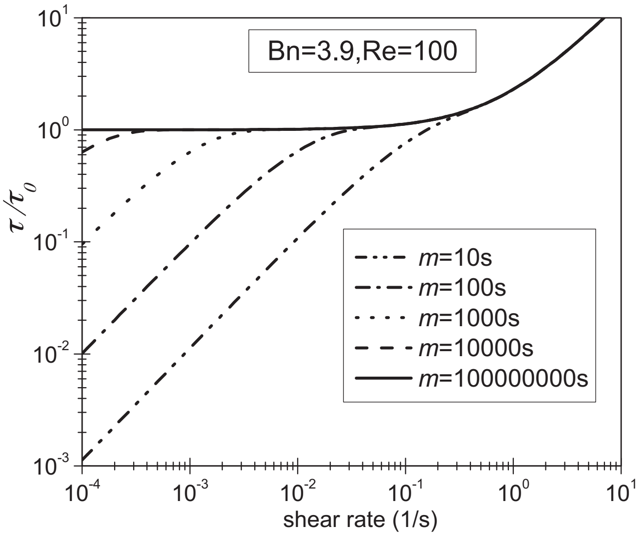

Fig. 1. The dimensionless shear stress against shear rate $\dot{\gamma}$ according to the modified Bingham constitutive equation (2) for several values of the exponent *m*.

posed as [23]

$$\tau = \tau_0 + \eta_p\dot{\gamma} \quad \text{if} \quad |\tau| > \tau_0, \tag{1a}$$

$$\dot{\gamma} = 0 \quad \text{if} \quad |\tau| \le \tau_0, \tag{1b}$$

where τ is the shear stress tensor, τ_0 is the yield stress, η_p is a constant plastic viscosity, and $\dot{\gamma}$ is the shear rate tensor. From these constitutive relations, it is known that when the magnitude of shear stress τ falls below τ_0, the material becomes a solid structure (unyielded). In order to avoid the inherent attribute of discontinuity in the viscoplastic model, Papanastasiou [22] proposed a modified equation that makes the shear stress vary continuously with the shear rate. This called regularization method makes the equation valid for both yielded and unyielded areas. With Papanastasiou exponential modification, the Bingham model becomes

$$\tau = \tau_0[1 - \exp(m\dot{\gamma})] + \eta_p\dot{\gamma}, \tag{2}$$

where *m* is the regularization parameter or the stress growth exponent, which controls the exponential growth of the stress. Ideal Bingham fluid can be mimicked for a large enough regularization parameter *m* to guarantee large apparent viscosity at vanishing rates of strain. Then from Eq. (2) the apparent viscosity of the Bingham fluid can be expressed as

$$\eta = \frac{\tau}{\dot{\gamma}} = \eta_p + \frac{\tau_0}{\dot{\gamma}}[1 - \exp(-m\dot{\gamma})], \tag{3}$$

where $\dot{\gamma}$ is the second invariant of the rate-of-strain tensor given by $\dot{\gamma} = \sqrt{2S_{\alpha\beta}S_{\alpha\beta}}$ and $S_{\alpha\beta}$ is defined as

$$S_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right). \tag{4}$$

Expressions mentioned above enable the shear stress to change continuously with the variation of shear rate.

To examine the effect of the value of exponent *m* on the approximation degree to ideal Bingham fluid, we present the shear stress against shear rate according to the modified Bingham constitutive equation (2) for several values of the exponent *m*. As shown in Fig. 1, the dimensionless shear stress τ/τ_0 approaches the ideal Bingham fluid (the solid line in the figure) as the value of *m* increases, which indicates that this equation can mimic the ideal Bingham fluid accurately for large enough *m*. However, in practical simulation, the value of *m* cannot be too large since it will result in numerical instability [24]. An intermediate value of *m* is usually chosen in the

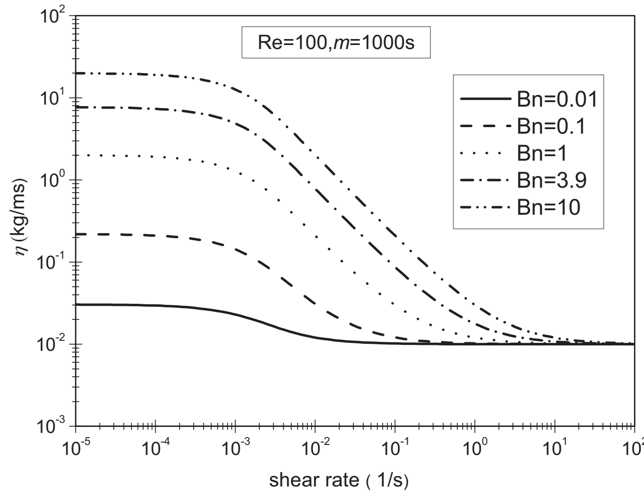

Fig. 2. The Bingham apparent viscosity against shear rate $\dot{\gamma}$ for different Bingham numbers at Re = 100 and *m* = 1000 s.

literature like *m* = 1000 s [5,17,24]. However, under the conditions of a fixed Reynolds number and exponent *m*, the approximation degree to the ideal Bingham fluid deviates obviously if we change Bingham number during the simulation as shown in Fig. 2, in which the apparent viscosity calculated by Eq. (3) varies against the shear rate for different Bingham numbers at fixed Re = 100 and *m* = 1000 s. We can see that when Bingham number is large, the apparent viscosity at lower shear rate is much larger than the viscosity at larger shear rate and hence the Bingham fluid can be mimicked very well. On the contrary, when Bingham number is small, the exhibited apparent viscosity difference between the lower shear rate and the larger shear rate is so small that the Bingham fluid is difficult to mimic. Therefore a much larger value of exponent *m* is suggested for this situation as shown in Fig. 3. From Fig. 3 we can see that the Bingham fluid can be mimicked well by adjusting *m* = 10,000–100,000 s at Re = 100 and a small Bingham number Bn = 0.01. Here the Reynolds number and Bingham number are defined as, respectively,

$$Re = \frac{\rho\bar{u}H}{2\eta_p}, \tag{5}$$

$$\mathrm{Bn} = \frac{\tau_0 H}{\eta_p\bar{u}}, \tag{6}$$

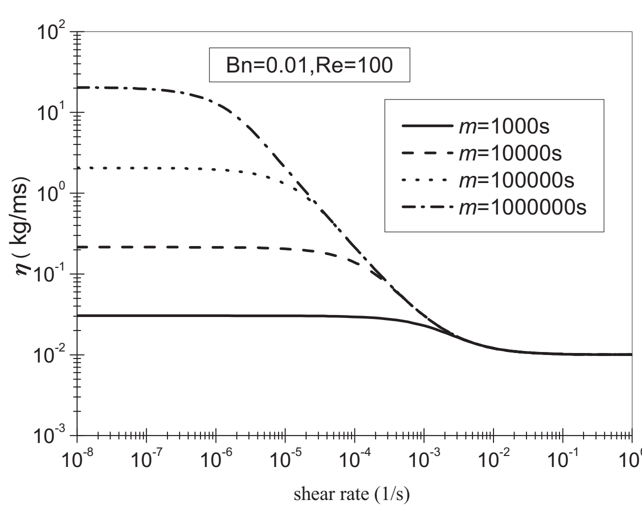

Fig. 3. The Bingham apparent viscosity against shear rate $\dot{\gamma}$ for different values of exponent *m* at a small Bingham number Bn = 0.01 and Re = 100.