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Research Paper

Efficient slope reliability analysis at low-probability levels in spatially variable soils

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ABSTRACT

Direct Monte Carlo simulation for the reliability analysis of slope stability with spatially variable soil properties suffers from a serious lack of efficiency when the probability of failure, p_f, is low (e.g., $p_f < 10^{-6}$). Based on the multiple response surfaces and Subset simulation, this paper proposes an efficient approach for the estimating of small probabilities of slope failure in spatially variable soils. An improved Cholesky decomposition technique is presented for the simulation of the globally non-stationary random fields of spatially variable soil properties in the multiple soil layers. Two slope examples are investigated to demonstrate the effectiveness of the proposed method. The efficiency of the proposed approach for parametric sensitivity analysis is also highlighted.

1. Introduction

It is widely recognized that soil properties vary spatially even within homogeneous layers due to depositional and post-depositional processes [43,25,38]. The spatial variability of soil properties has been widely accounted for in slope stability analysis. For example, Griffiths and Fenton [16] proposed a random finite-element method (RFEM) to investigate the effect of spatial variability of undrained shear strength on the probability of slope failure. Wang et al. [44] performed slope stability analysis considering spatially variable undrained shear strength using Subset simulation (SS). Ji et al. [20] proposed an EXCEL based first order reliability method (FORM) to analyze slope reliability in the presence of spatially varying shear strength parameters. Jha and Ching [19] employed a random finite element analysis (RFEA) to study the effect of spatial variation in the shear strength on the stability of undrained engineered slopes. Jiang et al. [22] applied a non-intrusive stochastic finite element method (NISFEM) to slope reliability analysis in spatially variable soils. Li et al. [28] developed a multiple response-surface method for probabilistic analysis to explore the influence of autocorrelation structure in the soil properties on the slope reliability. Additionally, many authors quantified the effect of the spatial variation in soil properties on the slope stability using direct Monte Carlo Simulation (MCS) [7,8,17,39,31,23,29].

It can be observed that significant advances have been made in the reliability analysis of soil slopes considering the spatial variability of soil properties. Table 1 summarizes the reliability analyses of soil slopes in spatially variable soils. These references are listed in a chronological order. It can be seen that the slope reliability problems with the probabilities of failure less than 10^{-4} have been rarely considered. It is well known that real probability of failure for safety–critical technical structures is often in the order of 10^{-4}–10^{-6} during the lifetime, or as low as 10^{-8} during one hour of operation [36,42]. In addition, the geometry of the slopes considered is very small as shown in Table 1. The height for the majority of slopes is less than 10 m. As reported in Jha and Ching [19], the height of many engineered slopes is larger than 10 m. Thus it is of significance to investigate the slope reliability problems involving relatively large slope geometries. For the slopes with relatively large geometries, a large number of random variables shall be discretized to guarantee the accuracy of random fields simulation [5,22], thereby resulting in high dimensional slope reliability problems. Theoretically, the direct MCS can be used for solving the high-dimensional slope reliability problems, but it could be really time-consuming especially when the probability of failure is low (e.g., $p_f < 10^{-6}$).

This study proposes to integrate multiple response surfaces with Subset simulation for the estimating of small probabilities

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Table 1
Summary of slope reliability analyses in spatially variable soils.

Paper ID	Authors	Slope height (m)	Reliability analysis methods	Probability levels
1	Li and Lumb [32]	10	FOSM	10^{-8}–10^{-3}
2	Griffiths and Fenton [16]	10	RFEM	10^{-1}
3	Cho [7]	10	MCS	10^{-5}–10^{-3}
4	Suchomel and Mašín [41]	17	FOSM	10^{-1}
5	Cho [8]	5, 10	MCS	10^{-3}
6	Huang et al. [17]	10	MCS	10^{-1}
7	Wang et al. [44]	10	Subset simulation	10^{-3}
8	Ji et al. [20]	5, 10	FORM	10^{-3}
9	Jha and Ching [19]	10	RFEA	10^{-4}–10^{-1}
10	Jiang et al. [22]	5	NISFEM	10^{-4}–10^{-1}
11	Salgado and Kim [39]	5.48, 5.78, 8.48, 4.35	MCS	10^{-4}
12	Li and Chu [31]	6	Surrogate model based-MCS	10^{-3}
13	Jiang et al. [23]	5, 10	Surrogate model based-MCS	10^{-4}–10^{-2}
14	Li et al. [29]	14.1	Surrogate model based-MCS	10^{-3}–10^{-1}

of failure in the limit-equilibrium analysis of slope stability with spatially variable soil properties. The effectiveness of the proposed approach is demonstrated using two examples, a two-layered cohesive slope and a real four-layered slope with heights of 24 m and 14.1 m, respectively. To achieve such a goal, the paper is organized as follows. In Section 2, the proposed approach comprising of the construction of multiple response surfaces, simulation of globally non-stationary random fields and estimating of small probabilities of failure is presented and implemented step by step. In Section 3, reliability analysis of two slope examples is carried out to illustrate the proposed approach. Finally, several conclusions are drawn from this study.

2. Multiple response-surface based Subset simulation approach

2.1. Construction of multiple response surfaces

The crucial step for reliability analysis of slope stability in the limit equilibrium framework is to search the critical slip surface and determine the corresponding minimum factor of safety ($FS_{\min}$). This is not easy when the spatial variability of soil properties is considered because the critical slip surface with the $FS_{\min}$ varies spatially which may induce numerous failure mechanisms existing in the slope [44]. Generally, the $FS_{\min}$ can be determined via deterministic slope stability analyses, but it is time-consuming for complex slope stability problems where the $FS_{\min}$ cannot be expressed explicitly as a function of input parameters. To reduce the computational cost, surrogate models such as response surface, Kriging and support vector machine are often adopted to construct the explicit function between the $FS_{\min}$ and input parameters (e.g., [45,49,47,46]). In this study, a quadratic polynomial without cross terms is used to establish the response surface between the factor of safety for each potential slip surface and input random variables. For instance, the j-th quadratic polynomial-based response surface is expressed as follows [45,47,31,28]:

$$FS_j(\boldsymbol{X}) = \sum_{i=1}^{N_c} \boldsymbol{a}_{ij} \Psi_{ij}(\boldsymbol{X}) = a_{1j} + \sum_{i=1}^{n} b_{ij} X_i + \sum_{i=1}^{n} c_{ij} X_i^2 \tag{1}$$

where $FS_j(\boldsymbol{X}), j = 1, 2, \ldots, N_p$, is the factor of safety for the j-th potential slip surface, N_p is the number of potential slip surfaces; $\boldsymbol{X} = (\boldsymbol{X}_1, \ldots, X_i, \ldots, X_n)^{\mathrm{T}}$ is the vector of input random variables in the physical space corresponding to those used to discretize the random fields; n is the number of input random variables which relates with that of random field elements (see Section 2.2); $\boldsymbol{a}_j = (a_{1j}, b_{1j}, \ldots, b_{nj}, c_{1j}, \ldots, c_{nj})^{\mathrm{T}}$ is the vector of unknown coefficients with a size of $N_c = 2n + 1$; $\Psi_{ij}(\cdot)$ is a quadratic polynomial expansion.

A sample design method using $(2n + 1)$ combinations proposed by Bucher and Bourgund [3] is then employed to determine the unknown coefficients in Eq. (1). The factor of safety for the j-th potential slip surface, $j = 1, 2, \ldots, N_p$, is first evaluated at $N_c = 2n + 1$ samples which are generated at the centroid of each random field element as below: $\left\{\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}\right\}$, $\left\{\mu_{X_1} \pm 2\sigma_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n}\right\}$, ..., $\left\{\mu_{X_1}, \mu_{X_2}, \ldots \mu_{X_i} \pm 2\sigma_{X_i}, \ldots, \mu_{X_n}\right\}$, ..., and $\left\{\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n} \pm 2\sigma_{X_n}\right\}$; μ_{X_i} and σ_{X_i} are the mean value and standard deviation of the i-th variable, respectively. In this way, a system of N_c linear algebraic equations can be established for the selected samples in terms of the unknown coefficients $\boldsymbol{a}_j$. Then, the unknown coefficients $\boldsymbol{a}_j$ are obtained by solving the system of equations directly. After that, a quadratic polynomial-based response surface is constructed for the j-th potential slip surface. Applying the similar method, multiple quadratic polynomial-based response surfaces (MQRSs) can be obtained and taken as the surrogate models of explicit functions between the factors of safety for N_p potential slip surfaces and the input random variables. Note that the obtained MQRSs do not involve the realizations of random fields, thus not rely on the statistics (e.g., mean, coefficient of variation, COV, and marginal distribution) of the soil properties. After that, the performance function for slope reliability analysis can be derived as follows:

$$G(\boldsymbol{X}) = FS_{\min}(\boldsymbol{X}) - 1.0 = \min_{j=1,2,\ldots,N_p} FS_j(\boldsymbol{X}) - 1.0 \tag{2}$$

where $\min_{j=1,2,\ldots,N_p} FS_j(\boldsymbol{X})$ is the minimum value of the MQRSs for factors of safety at a given realization of random fields in the physical space. In this way, the values of $G(\boldsymbol{X})$ in slope reliability analysis can be directly obtained by substituting the realizations of random fields into Eqs. (1) and (2), without performing deterministic slope stability analyses again. Therefore, the computational efficiency for determining the values of the $FS_{\min}$ and $G(\boldsymbol{X})$ at each realization of random fields is greatly improved.

2.2. Simulation of globally non-stationary random fields

A random field is called stationary or weakly stationary if the following conditions are jointly met [43,38,39,6,28]: (1) the mean and variance are the same at every point within the field. (2) an autocorrelation function governs the degree of correlation between the residuals of any two points in the domain, which depends only on the distance between any two points within the random field not on absolute locations. (3) the probability density function for samples of the same size is independent of the absolute locations. However, the random fields of soil properties in geotechnical practice often cannot jointly meet the above mentioned conditions. For instance, the means and standard deviations of soil parameters