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Research Paper

A fractal model based on a new governing equation of fluid flow in fractures for characterizing hydraulic properties of rock fracture networks

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ABSTRACT

This study presents a fractal length distribution model of fractures in discrete fracture networks (DFNs), adopting a fractal dimension D_f that represents the geometric distribution characteristics of fractures and another fractal dimension D_T that represents the tortuosity of fluid flow induced by surface roughness of single fractures in DFNs. A new governing equation for fluid flow in single fractures based on the cubic law was incorporated into this fractal model. Fluid flow in 1290 DFNs with different geometric characteristics of fractures and side lengths was simulated and their equivalent permeability was calculated. The results show that the values of *a*, which is the power law exponent of the fracture size distribution, calculated by the proposed fractal model are consistent with those reported in similar previous studies. The flow rate of a DFN changes proportionally with e^{6-D_T} where *e* is the aperture, which agrees better with the in-situ measurements reported in literature than the prediction of classical cubic law (e^3). The equivalent permeability of DFNs is more sensitive to the random number utilized to generate the fracture length than the ones used to generate the orientation and center point of fractures. With the increment of D_f, the size of the representative elementary volume (REV) decreases. When the size of a DFN is larger than the REV, the variation of equivalent permeability induced by the random number holds constant. When $D_f < 1.5$, fluid flow in a DFN is dominated by the relatively small fractures with their lengths shorter than the side length of the DFN. With increasing D_f, fluid flow becomes more dominated by the longer fractures, especially the ones cutting through the models.

1. Introduction

Fluid flow in fractured rock masses is governed by connected conductive fractures, due to their significantly higher permeability comparing with the rock matrix. In the past few decades, the discrete fracture network (DFN) modelling techniques have been developed to establish discontinuous models of fractured rock masses based on the assumption that fluid only flows in the connected conductive fractures with negligible rock matrix permeability [38,37,61,42,43,3,32,35,34]. Field mapping provides information of geometric characteristics of fractures for establishing DFN models, which could then be used for assessing the fluid flow characteristics numerically. A number of parameters are involved in the description of fracture geometries in rock masses (e.g., length, aperture, and dip angle). It is still a challenging and time consuming task to accurately obtain the geometric information of each single fracture in a field at both the macro-structural (i.e., geometry of the fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [44,16,59,33]. As an alternative, a number of mathematical expressions have been proposed to represent the characteristics of fracture distributions and to describe fluid flow behavior in single fractures with complex void geometries. Jafari and Babadagli [20,21] have showed that the density and length of fractures are the two most critical parameters for calculating the equivalent fracture network permeability (EFNP). De Dreuzy et al. [11–13] have semi-empirically found that the fracture lengths follow a power law distribution, yet the geologists have to heavily account

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Nomenclature

a	power law exponent
a''	the first regression parameter
b''	the second regression parameter
c''	the third regression parameter
d_m	mass density of fractures
D_f	fractal dimension of fracture backbones
D_T	fractal dimension that represents fracture surface roughness
E	Young's modulus
e	hydraulic aperture of a fracture
e_{avg}	average displacement in opening-mode fractures
e_{max}	maximum opening displacement
g	gravitational acceleration
i	fracture id
K	equivalent permeability of a fracture network
K_{1c}	intrinsic fracture toughness of the material
L	straight length of a fracture
l	trace length of a fracture
l_{max}	maximum trace length
l_{min}	minimum trace length
L_T	side length of a fracture network corresponding to a total fracture number of N_t
L_n	side length of an arbitrary fracture network
L_t	tortuous length of a fracture
N_t	total number of fractures in a network
N'_t	fracture number corresponding to a side length of fracture network of L_n
Q	flow rate
R	random number
u_{max}	maximum shearing displacement
V_M	maximum variance of equivalent permeability
W	width of a fracture
Δh	hydraulic head difference
∇h	hydraulic pressure gradient

Greek letters

α	coefficient of proportionality in power law
α'	coefficient of proportionality in trace length – aperture relationship
γ	coefficient related to the mechanical properties of surrounding rocks
μ	dynamic viscosity
ν	Poisson's ratio
ρ	fluid density

for the length of each fracture to determine the power law exponent. Other studies have shown that the geometric distributions of fractures in fracture networks, especially the length of fractures, exhibit fractal characteristics, which presented a promising approach to quantitatively estimate the characteristics of fracture distributions. (e.g., [57,15,48,25]).

Barton and Larsen [6], La Pointe [30], Barton and Hsieh [5] found that natural fracture patterns exhibit fractal characteristics based on statistical analysis of natural cracks and fractures. Babadagli [1] mapped natural fracture patterns of 2-D fracture networks of geothermal reservoirs at different scales. They observed that the fracture networks exhibit scale-invariant properties, however, fractal dimensions might significantly differ when the mass dimensions were measured by different methods. Bagde et al. [2] calculated the fractal dimensions of blasted fragments and in situ rock blocks using size distribution curves. They concluded that the change of fractal dimension is nominal beyond a uniaxial compressive strength (UCS) value of 20 MPa, while there is a sharp increase in fractal dimension for rock mass rating (RMR) greater than 40. Zhao et al. [62] found that the random distribution of fractures in a geologic mass agrees well with the fractal law. Their observations and statistics based on the data of three sites demonstrated that fracture distribution of each group, classified by the strike of the strata, still follows the fractal law, although the fractal dimension varies with different strikes to some extent. Zheng and Yu [64] established a fractal permeability model for gas flow through dual-porosity media by embedding fractal-like tree networks. Their calculation results showed that the porous matrix can be seen as a gas storage medium with negligible contributions to gas flow, and the permeability of a dual-porosity medium may primarily be controlled by the fractures. Jafari and Babadagli [22] analyzed the influences of fracture network characteristics (density, length, orientation, connectivity, and aperture) on permeability using different calculation methods of the fractal dimension. A nonlinear multivariable regression was derived to estimate the equivalent fracture permeability based on five independent variables. Jafari and Babadagli [23] later presented the relationship between percolation-fractal properties and permeability of 2-D fracture networks. They found that the fractal dimension of fracture lines obtained using the box counting method yields a more accurate estimation, comparing with the fractal dimensions of intersection point, connectivity index, and scanning lines in X- and Y-directions, for EFNP. Kruhl [28] reviewed the applications of fractal-geometry techniques in the quantification of complex rock structures considering the scale effect, inhomogeneity, and anisotropy of rock masses. Miao et al. [41] derived an analytical expression for permeability of fractured rocks involving fractal dimensions for representing the fracture area, area porosity, fracture density, the maximum fracture length, aperture, fracture azimuth, and fracture dip angle. In most previous 2-D DFN models, the fractures were typically treated as straight lines, without considering their geometric properties in the out-of-plane orientations. The hydro-mechanical properties of single rough rock fractures have been extensively studied experimentally and numerically (e.g., [7,66,58]), however, fractures in DFN models are still treated typically as parallel-plate models to allow the application of the cubic law, in which the flow rate is proportional to the cube of aperture by assuming a unit value of fracture width in the out-of-plane orientation. There are limited works that have taken into account the effects of surface roughness in DFN models. For example, Zhao et al. [63] studied the effects of fracture surface roughness on the macroscopic fluid flow in 2-D fracture networks, without considering the out-of-plane geometry of fractures. Most previous studies (e.g., [6,62,22]) primarily focused on the analysis of field mapping data to estimate the fractal distribution characteristics of the fracture length, rather than theoretically deriving some expressions for fracture length distributions. Miao and Yu [41] has stressed the influence of tortuosity on the total equivalent permeability of fracture networks, yet their model assumed that each fracture cuts through the model without considering the orientation and intersection of fractures, which, to some extent, may deviate from the real geometries of rock masses. To solve these problems, in a previous study, we have theoretically developed a fractal model in which the length distribution of fractures follows