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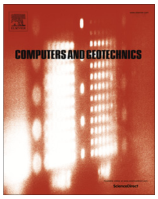

Research Paper

Hydro-elasto-plastic modelling with a solid/fluid transition ☆

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This paper deals with a new model for solving coupled hydromechanical problems, based on an existing unified model. Firstly, the unified model for granular media, describing solid and fluid states with the transition between them, is briefly presented and extended to the unsaturated domain using Bishop's effective stress. Secondly, an adapted stress–strain relationship is derived from modified Van Genuchten–Mualem's water retention curves. On this basis, a finite element formulation with Lagrangian integration points in a visco-elasto-plastic framework is proposed and implemented in a FEMLIP tool. Finally, the formulation is validated with reference to several benchmarks. The results and analysis show its reliability, and by means of FEMLIP, more complex and realistic problems are expected to be solved in the field of natural risks.

1. Introduction

In the early history of soil mechanics, researchers generally focused on the study of dry or saturated soils, so most soil mechanics theories can only apply to these conditions. As knowledge improved, increasing phenomena were observed related to the unsaturation of soils. Actually, as is well known, unsaturation gives soil several significant features such as enhanced strength, increasing fragility and plastic collapse along with wetting processes under certain stress levels. In the past few decades, there has been an increasing interest in the study of coupled hydromechanical problems in geomaterial porous media. Alonso et al. [4] were the first to provide a complete elasto-plastic framework for unsaturated soil; then a large number of constitutive models, giving more or less schematized stress–strain relationships, were established [5–7]. In recent years, more highly developed models have included suction–saturation relationships [34], such as the models proposed by Gallipoli et al. [8], Wheeler et al. [11], Tarrantino et al. [12], and Sheng et al. [35]. All these models have taken into account the water retention curves with a hydraulic hysteresis.

The above-mentioned constitutive models elaborate the solid-like behaviour of geomaterials. However, saturated loose geomaterials generally exhibit a fluid-like behaviour with a burst of kinetic energy in the postfailure stage. An example is the onset and propagation of a flow-type landslide. To describe such geomaterials comprehensively and completely, a unified model that consists of appropriate solid-like and fluid-like models and a criterion of solid–fluid transition is required. Recently, a unified model, in which solid-like behaviour and fluid-like behaviour are described by the PLASOL elasto-plastic model [9] and the Bingham viscous model, respectively, while Hill's second-order work criterion [13] is chosen as the criterion of transition, has been established [10,28]. In this paper, this model will be extended to unsaturated conditions.

A such simulation of complete solid–fluid behaviours requires solving large transformation problems. Several mesh-free methods are considered herein, such as smoothed particle hydrodynamics (SPH) [42] and the material point method (MPM) [43]. Because it has the advantage of tracking the history of the variables involved in elasto-plasticity and describing large transformations, the finite element method with Lagrangian integration points (FEMLIP), developed from "Particle-in-Cell" method [33], is used in this paper.

This paper proceeds as follows. In Section 2 the unified model is briefly introduced, and the adapted effective stress and modified Van Genuchten's water retention curves are specified. Section 3 establishes the derivation of stress–strain relations and the finite element formulation. The global visco-elasto-plastic constitutive relation in a hydromechanical framework presents all the informations for the computations in Ellipsis, the code based on the FEMLIP method. Section 4 gives three benchmarks to discuss and validates the readability of Ellipsis for solving hydromechanical problems. Finally, in Section 5, conclusions are drawn and future prospects are discussed.

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Nomenclature

Exponents and index

$\overline{(.)}$	net value
$(.)'$, $(.)_{eff}$	effective value
$(.)_0$, $(.)^0$	initial value
$(.)_f$	final value
$(.)^t$	current value
$\dot{(.)}$	temporal differential
$(.)^T$	transposition
$(.)_{tot}$	total value
$(.)_v$	viscous value
$(.)_e$	elastic value
$(.)_p$	plastic value
$(.)_n$	numerical value

Scalars

c	cohesion
ϕ_c	mobilised friction angle under triaxial compression paths
ϕ_e	mobilised friction angle under triaxial extension paths
$J_{1\sigma'}$, $J_{2\sigma}$, $J_{3\sigma}$	effective stress tensor invariants
d^2w	local second-order work
D^2W	global second-order work
ω_i	numerical weight of integration point i
J_i	determinant of Jacobian matrix
s_y	Bingham yield stress
η	dynamic viscosity
χ	parameter of Bishop's effective stress
a_χ, n_χ	parameters of χ
u_a, u_w	air pressure and water pressure
s	suction
s_{aev}	air entry value
P_{atm}	atmospheric pressure
Sr	degree of saturation
θ	water content
a_v, n_v	parameters of Van Genuchten–Mualem's WRCs
k, λ	constant material parameters
n	porosity
t	time
q	Darcy's velocity normal to boundary
S	boundary subjected to force
S_w	boundary subjected to water flux
V	volume considered
g	gravity acceleration
ρ	density
ρ_w	water density
K	bulk modulus
μ	elastic shear modulus
v	Poisson's coefficient
p	total mean pressure
k_r	relative hydraulic conductivity

Vectors

$\boldsymbol{V_k}$	Darcy's velocity
$\boldsymbol{f}$	boundary force vector
$\boldsymbol{b}$	body force vector
$\boldsymbol{b_w}$	body force vector of pore water
$\boldsymbol{U}$	nodal displacement vector
$\boldsymbol{U_w}$	nodal water pressure vector
$\boldsymbol{N_w}$	water pressure shape function

Tensors

$\boldsymbol{D^{ep}}$	elasto-plastic matrix
$\boldsymbol{D^v}$	viscous matrix
$\boldsymbol{D^{ve}}$	visco-elastic matrix
$\boldsymbol{\sigma}$	total stress tensor
$\boldsymbol{\varepsilon}$	strain tensor
$\boldsymbol{\tau}$	deviatoric stress tensor
$\boldsymbol{e}$	deviatoric strain tensor
$\boldsymbol{k}$	unsaturated permeability tensor
$\boldsymbol{k_s}$	saturated permeability tensor
$\boldsymbol{N}$	velocity shape function

2. A unified solid–fluid model incorporating hydromechanical coupling

Fine soils are known to exhibit elasto-plastic behaviour as solids and viscous behaviour as fluids alternatively. To model an entire process of the geomaterial loss of stability, an existing unified model in 3D, describing the solid, fluid states with a transition between them is taken into account. For greater detail on this model, the reader can refer to Prime et al. [2,10,28]. To solve hydromechanical problems, this model is extended here for partially saturated conditions by introducing Bishop's effective stress and a water-retention model.

2.1. Elasto-plastic model with a hydromechanical coupling

2.1.1. Bishop's effective stress

The discussion on the choice of the proper stress variables in unsaturated conditions is still open. For greater detail, the reader can refer to [22,23]. In this paper, Bishop's effective stress, expressed as follows, is used in the elasto-plastic model to simulate the solid behaviour of unsaturated soils:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - u_a\boldsymbol{m} + \chi(u_a - u_w)\boldsymbol{m} \tag{1}$$

where $\boldsymbol{\sigma}'$ is the effective intergranular stress vector, $\boldsymbol{\sigma}$ the total stress vector, u_a the isotropic air pressure, and u_w the isotropic water pressure. $\boldsymbol{m^T} = (1,1,1,0,0,0)$ and $s = u_a - u_w$ is the suction component. A six-component vectorial rotation is used for σ and σ'.

Determining χ is a delicate point. Besides the most common formulation $\chi = Sr$ [24], many researchers have studied and proposed several expressions for this parameter, such as Khalili and Khabbaz [25] and Alonso et al. [26]. Arairo et al. [27] proposed the following expression on the basis of Alonso's work:

$$\chi = \left(1 + \left(\frac{a_\chi s}{P_{atm}}\right)^{n_\chi}\right)^{\frac{1}{n_\chi}-1} \tag{2}$$

where a_χ and n_χ are parameters defined to ensure that the value of χ is always located between two boundary water retention curves for a given suction value. By adjusting the two parameters, many features of unsaturated soils can be described including plastic collapse in the wetting process [27]. Let us note, however, that the most recent advances in unsaturated granular media have shown the tensorial nature of χ [38,39].

2.1.2. PLASOL constitutive relation

In the solid stage, the behaviour of unsaturated geomaterials is described by the PLASOL non-associated elasto-plastic model by means of Bishop's effective stress mentioned above. This model, appropriate to deal with a wide range of diversified soils, was developed at Liege University; more detailed information is