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Bounding surface model for soil resistance to cyclic lateral pile displacements with arbitrary direction

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ABSTRACT

The development of a two-surface elastic–plastic bounding surface *P–Y* model for cyclic lateral pile motions is described. The kinematic-hardening model is applicable to the analysis of pile foundations subjected to loading with arbitrary azimuths relative to the pile axis. The model realistically captures the hysteretic energy damping associated with dynamic loading of subsea foundations through physically correct plastic mechanisms and provides results consistent with those observed in physical tests including cyclic loading. Its performance is demonstrated in element states of stress and in pile foundation analyses. The development based on the incremental theory of plasticity results in more robust solutions than may be obtained using alternative elastic, variable moduli and deformation plasticity formulations.

1. Introduction

The development of an elastic–plastic bounding surface *P–Y* (BSPY) model for three dimensional pile response simulation is presented for an undrained cohesive soil. The work builds on a previous model development for planar motions [1]. The model response includes a viscous component to characterize rate effects and small amplitude damping. The ability to adequately reproduce the soil hysteretic response is the motivation for development of the model. Its implementation through the concepts of work-hardening plasticity results in numerically stable solutions for arbitrary load paths.

The BSPY model is implemented in an overlay fashion [2] with two components: an elastic–plastic bounding surface component, and a viscous component. The model can be calibrated to replicate the response of traditional *P–Y* models used for monotonic loading in cohesive soils. Experimental [3], numerical [4] and limit analysis [5] studies imply that the traditional industry *P–Y* models for cohesive soils likely underestimate the strength and stiffness of soil resistance to lateral pile motions. The BSPY model offers the flexibility to account for alternative characteristics of the lateral soil resistance to monotonic and cyclic load responses. The model performance is illustrated for both element conditions and in finite element applications. Its characteristic responses are compared to responses observed in physical tests with cyclic loading.

2. BSPY model components

The bounding surface implementation follows the theoretical formulation presented by Dafalias and Popov [6,7], Dafalias and Herrmann [8] and Krieg [9]. The concept is presented in two-dimensional form in Fig. 1a. Fig. 1b shows the form of the model viscous component. The two components are paired in an overlay model formulation. The bounding surface model includes two kinematic hardening surfaces, a yield surface defining an elastic zone and a 'bounding surface', as well as a perfectly-plastic limiting strength surface. The combination of the two kinematic surfaces allows representation of cyclic Bauschinger effects. In the following, the model is sometimes described in terms of 'stress' and 'strain', although in reality it is implemented in terms of displacement and a resistance in force/length. The descriptions are meant to be interchangeable.

The aforementioned authors produced bounding surface models with a range of features and applications (metals and soils) with kinematic and mixed-hardening surfaces. The model here is formulated in the conventional manner for incremental plasticity models and the implementation closely follows that described by Krieg [9] for metals with respect to the rules regarding surface translation. The two-dimensional form results in some simplifications compared to a six-dimensional stress space. The incremental elastic–plastic stiffness C^{ep} relating the resistance dP and the displacement dy is of the form

$$dP = C^{ep} \cdot dy \quad (1)$$

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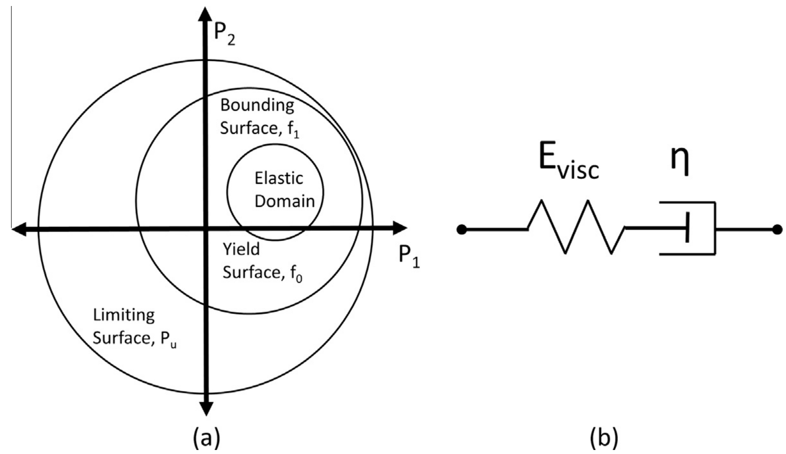

Fig. 1. BSPY model; (a) yield and bounding surfaces, (b) viscous component.

The elastic stiffness C in the model is isotropic with the response in the two orthogonal directions uncoupled

$$C = E \cdot I \tag{2}$$

where E is the elastic stiffness and I is a 2×2 identity matrix. The plastic kinematic surfaces in two dimensions have the form

$$f_n = (P - \propto_n) \cdot (P - \propto_n) - \kappa_n^2 = 0; \quad n = 0, 1 \tag{3}$$

where κ_n is the radius of the surface, P are the stresses, and $\propto_n$ are hardening parameters representing the center of the surfaces. The translation of the yield surface is governed by a hardening modulus dependent on the distance δ between the stress point on the yield surface f_0 and an image point on the bounding surface f_1, and is along the unit vector μ between the two points. The image point on the bounding surface is that with the same normal direction as the loading point on the yield surface. When loading is on the bounding surface, the direction of translation is normal to the bounding surface and both surfaces translate together. Associated flow rules are used so that the plastic deformations are in the direction of the normal to the loading surface. The elastic–plastic stiffness is then

$$C^{ep} = C - \frac{C \frac{\partial f}{\partial P} \frac{\partial f}{\partial P}^T C}{2\kappa H_m \frac{\partial f}{\partial P}^T \mu + \frac{\partial f}{\partial P}^T C \frac{\partial f}{\partial P}} \tag{4}$$

In Eq. (4), f and κ are those associated with the outermost surface f_n on which loading is occurring. The denominator of Eq. (4) takes advantage of the fact $\partial f / \partial \alpha = -\partial f / \partial P$, according to the mathematical form of the surfaces in Eq. (3), otherwise the first term of the denominator would include a partial derivative with respect to α in place of the partial derivative with respect to P. The plastic constitutive response results in a stress-induced anisotropic behavior where the stiffness and resistance in two orthogonal directions are coupled through the plastic constitutive relationship. The plastic hardening modulus H_m can be defined with some flexibility, for example

$$H_m = f(H_0, H_1, \delta, \overline{y^p}) \tag{5}$$

where H_0 and H_1 are the hardening moduli for the yield and bounding surfaces, respectively, $\overline{y^p} = \int \overline{dy^p}$, dy^p are the incremental plastic displacements, and $\overline{dy^p} = (dy^p \cdot dy^p)^{1/2}$. The dependency of the material hardening on the plastic history allows for a degradation of observed elastic–plastic stiffness as a result of cyclic loading. In the present case, the plastic hardening modulus is reduced by a factor F_m

$$F_m = 1 - F_1(1 - \exp(-\overline{y^p} F_2)) \tag{6}$$

where F_1 and F_2 are model parameters. The BSPY hardening response is of course a phenomenological simplification of the actual soil response, which is likely associated with pore pressure build-up and migration of stress paths to states more susceptible to plastic deformation (e.g., [8,10]). Such responses are not necessarily associated with material softening and may be, at least partially, recoverable.

The specific form of H_m adopted here is

$$H_m = (H_0 \bar{\delta} + H_1) \cdot F_m; \quad \bar{\delta} = \delta / (2\kappa_1 - \delta) \tag{7}$$

where κ_1 is the radius of the bounding surface. When $\delta = 0$, H_m is directly proportional to the outer surface value, and when δ is large the plastic modulus increases and the constitutive response tends toward the elastic response. The restriction here is that $2\kappa_1 - \delta > 0$, which implies the radius of the yield surface is nonzero. The magnitude $\bar{\alpha}_{max}$ of the maximum excursion of the bounding surface center $\propto_1$ from the origin of the stress space is limited so that the stress space origin is contained within the bounding surface. The limiting excursion combined with the fixed radius of the bounding surface gives the ultimate resistance of the model, which in effect is a third surface P_u in the model (Fig. 1). The translation of surfaces are governed by a hardening rule of the form

$$d\alpha = H \beta \overline{dy^p} \tag{8}$$

When loading is on the yield surface, $d\alpha_0$ is computed with $H = H_m$, $\beta = \mu$ is as described above, and $d\alpha_1$ is computed with $H = F_m H_1$ with β normal to the image point on the bounding surface. When loading is on the bounding surface, both surfaces are in contact and translate together, with $H = H_m$ (note $\delta = 0$) and β is normal to the bounding surface. If loading reaches the limiting surface P_u, then the response of the model during further loading is according to its perfectly plastic response and the hardening parameters $\propto_n$ of the kinematic surfaces are updated to result in a unique gradient direction for all three surfaces at the point of loading.

The linear viscous component (Fig. 1b) provides a convenient mechanism accounting for viscous damping at small displacements. Proper selection of the model components might account for a rate stiffening effect, but care should be taken that the results are reasonable. The formulation adopted is that described by Silva [11], which requires the definition of a viscous stiffness E_{visc} and a viscous damping parameter η. The total BSPY resistance P_{TOT} including the response described by the bounding surface stress P and the viscous component P_V is then

$$P_{\text{TOT}} = P + P_V \tag{9}$$