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Two-dimensional laminar flow of a power-law fluid across a confined square cylinder

Akhilesh K. Sahu [a], R.P. Chhabra [a,*], V. Eswaran [b]

[a] Department of Chemical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India
[b] Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India

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ABSTRACT

Two-dimensional laminar flow of power-law fluids past a long square cylinder confined in a planar channel is investigated numerically for the range of conditions as $60 \le Re \le 160$, $0.5 \le n \le 1.8$ and $\beta = 1/6$, $1/4$, and $1/2$. A semi-explicit finite volume method is used on a non-uniform collocated grid arrangement. The third order QUICK scheme and the second-order central difference scheme are used to discretize the convective and diffusive terms respectively. Depending upon the value of blockage ratio, power-law index and Reynolds number, the nature of flow in the above range of conditions is either steady or unsteady (periodic in time). An increase in the blockage ratio delays the onset of vortex shedding to higher Reynolds number in both shear-thinning and shear-thickening fluids whereas it advances the occurrence of the quasi-periodicity in flow to lower Reynolds numbers in shear-thinning fluids. Extensive numerical results are presented to elucidate the effects of blockage, power-law index and Reynolds number on the drag coefficient, stream function, vorticity, Strouhal number and amplitudes of drag and lift coefficients in the unsteady flow regime.

1. Introduction

The flow of fluids past cylinders of different cross-sections (circular, square, elliptic for instance) is a classical problem within the domain of fluid mechanics. Such model studies have received impetus from both theoretical considerations such as to advance our understanding of the wake phenomena and vortex shedding, etc. as well as pragmatic considerations such as reliable prediction of time-averaged hydrodynamics forces (lift and drag coefficients) is needed in numerous instances. For instance, the flow over a long cylinder is encountered in tubular and pin type heat exchangers, thermal processing of food stuffs like potato and carrot chips, etc. Bluff bodies of various shapes are also used as flow dividers to from weldlines in polymer forming operations. Consequently, over the years, a voluminous body of knowledge has accrued on the flow of Newtonian fluids past cylinders of various cross sections. The bulk of the literature pertains to circular cylinders, followed by that for square, elliptic and rectangular cylinders. Most of the literature up to 2003 is thoroughly reviewed in the two-volume set of Zadravkovich [1,2] whereas the literature pertaining to the vortex shedding has been summarised by Williamson [3]. Indeed, even for the simplest shape of a circular cylinder which is free from geometrical singularities, the flow exhibits a rich variety of phenomena depending upon the nature of the mainstream flow (uniform shear, confined/unconfined for instance), type of fluid (Newtonian or non-Newtonian), aspect ratio of the cylinder (length to diameter ratio) and the characteristic Reynolds number of the flow. Even for the simplest case of the unconfined uniform flow of Newtonian fluids past an infinitely long circular cylinder, depending upon the value of the Reynolds number, the flow can be 2D steady without separation, 2D steady with separation, 2D unsteady with laminar vortex shedding, to become turbulent and 3D as the Reynolds number is progressively increased. Due to the inherently different nature of the underlying physical processes, global parameters like drag and lift coefficient, Nusselt number, etc. scale differently with Reynolds number in each flow regime. Furthermore, the transitional value as well as scaling is also influenced by the type of the fluid, severity of confinement, end effects, and nature of the far flow field [1,2,4]. All in all, adequate information is now available over the range of interest for Newtonian fluid flow past a circular cylinder. A reasonable body of knowledge (experimental and numerical) is also available for Newtonian fluid flow past a cylinder of square cross-section in unconfined configuration [5–15], and for a confined cylinder [16–21]. Broadly, currently available results encompass the laminar vortex shedding regime. Similarly, a limited body of information is also available for cylinders of elliptic cross-section, e.g., see [22,23].

Many multiphase and polymeric materials encountered in numerous industrial settings (especially in food and pharmaceutical, polymer and process engineering applications) display a

* Tel.: +91 512 2597393; fax: +91 512 2590104.
E-mail address: chhabra@iitk.ac.in (R.P. Chhabra).

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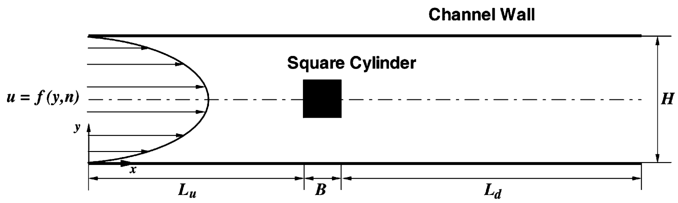

Fig. 1. Schematics of the flow around a square cylinder confined in a channel.

range of non-Newtonian characteristics including shear-thinning, shear-thickening, visco-elasticity, yield stress, etc. [24]. Perhaps the commonest non-Newtonian characteristic is of shear-dependent viscosity which is frequently approximated using the simple power-law fluid model. A few studies are available on the flow of power-law fluids past a circular cylinder, most of which pertain to the steady flow regime; all of these have been summarized recently [25,26]. Even fewer studies are available on the flow of power-law fluids past an unconfined square cylinder [25–28]. There have been only two studies of the effect of planar confinement on momentum characteristics of a square cylinder immersed in power-law fluids which are briefly described here. This problem has been initially investigated by Gupta et al. [29] for a single value of the blockage ratio ($\beta = 1/8$) over the range of conditions as $5 \le Re \le 40$, $5 \le Pe \le 400$ and $0.5 \le n \le 1.4$. Later on, Dhiman et al. [30] extended this work to three different blockage ratios ($\beta = 1/8$, 1/6 and 1/4) over a wider range of conditions with relatively finer grids than used by Gupta et al. [29]. Subsequently Dhiman [31] extended this work to study forced convective heat transfer from a square cylinder to shear-thickening fluids ($n > 1$) for a single blockage ratio of 1/8. Thus very little information is available in the literature on the effect of blockage for the flow of power-law fluids past a square cylinder, especially beyond the steady flow regime. This study aims to explore the effects of blockage ratio, β, and the power-law index, n, on the flow across a confined square cylinder in the unsteady flow regime. In particular, numerical results are presented here for the following ranges of conditions: $0.5 \le n \le 1.8$, $\beta = 1/6$, 1/4, and 1/2 and $60 \le Re \le 160$. However, similar to our previous work here also, depending upon the value of both the power-law index ($n < 1$) and blockage ratio (β), the maximum value of the Reynolds number is chosen such that the flow is truly fully periodic in time.

2. Problem statement and mathematical formulation

The physical problem considered in this study is the flow of an incompressible power-law fluid around a square cylinder placed symmetrically in a horizontal channel (height h and length L) as shown schematically in Fig. 1. The flow is assumed to be unsteady, 2D and laminar, and the inlet velocity profile is assumed to be fully developed. The ratio of the size of the cylinder, B, to the height of the channel, H, defines the blockage ratio, β. The blockage ratio is varied by changing the height of the channel, H. The choice of the upstream length L_u and the downstream length L_d are based on previous studies [20,30]. In the governing equations, the space coordinates, velocities, time, pressure and viscosity are rendered dimensionless by using the size of square cylinder (B), the maximum velocity at channel inlet (u_{max}), the characteristic time (B/u_{max}), the characteristic pressure ρu_{max}^2 and the characteristic viscosity $m\left(u_{max}/B\right)^{n-1}$ respectively. For an incompressible, 2D and laminar flow, the integral forms of the continuity, the x- and y-components of Cauchy's equation in their dimensionless form are given below.

Continuity:

$$\int_S \mathbf{V} \cdot dS = 0 \tag{1}$$

x-Momentum:

$$\frac{\partial}{\partial t}\int_\Omega u\, d\Omega + \int_S u\mathbf{V}\cdot \mathbf{dS} = -\int_S p\hat{i}\cdot \mathbf{dS} + \frac{2}{Re}\int_S (\eta\epsilon_{ii}\hat{i} + \eta\epsilon_{ij}\hat{j})\cdot \mathbf{dS} \tag{2}$$

y-Momentum:

$$\frac{\partial}{\partial t}\int_\Omega v\, d\Omega + \int_S v\mathbf{V}\cdot \mathbf{dS} = -\int_S p\hat{j}\cdot \mathbf{dS} + \frac{2}{Re}\int_S (\eta\epsilon_{ji}\hat{i} + \eta\epsilon_{jj}\hat{j})\cdot \mathbf{dS} \tag{3}$$

where the dimensionless Reynolds number is defined as

$$Re = \frac{B^n u_{max}^{(2-n)}\rho}{m} \tag{4}$$

Here m and n are the constants of the power-law viscosity model. In Eqs. (2) and (3), $\mathbf{dS}$ is given by $\hat{\mathbf{n}}_\mathbf{s}\, dS$ ($\hat{\mathbf{n}}_\mathbf{s}$ is the unit normal vector to the surface dS) and $\hat{i}, \hat{j}$ are the unit vectors in the x- and y-directions, respectively. For an incompressible fluid, the components of the extra stress tensor are related to the components of the rate of deformation tensor, ϵ as

$$\tau_{ij} = 2\eta\epsilon_{ij} \tag{5}$$

where $\epsilon_{ij} = (1/2)(\partial_j V_i + \partial_i V_j)$. The non-Newtonian viscosity behaviour of the fluid is modeled here by the power law model. This model expresses the apparent viscosity $\eta'(\dot{\gamma}')$ (ratio of shear stress to shear rate) as a function of the shear rate ($\dot{\gamma}'$) as follows:

$$\eta' = m\dot{\gamma}'^{(n-1)} \tag{6}$$

where m and n are the power-law consistency index and flow behaviour index respectively. Finally, the nondimensional power-law viscosity is calculated as follows:

$$\eta = (2\epsilon_{ij}\cdot\epsilon_{ji})^{(n-1)/2} \tag{7}$$

The boundary conditions (dimensionless) for this flow are written as follows:

- At inlet, the flow is assumed to be fully developed, i.e.,

$$u = \left(1 - \left|(1 - 2\beta y)\right|^{(n+1)/n}\right) \tag{8}$$

- At upper and lower channel walls, the usual no-slip condition is prescribed, i.e.,

$$u = 0, \qquad v = 0 \tag{9}$$

- On the surface of the square cylinder, the no-slip condition is used, i.e.,

$$u = 0, \qquad v = 0 \tag{10}$$

- At the exit boundary, the Orlanski condition [32] is employed which is expressed as

$$\frac{\partial\phi}{\partial t} + U_c\frac{\partial\phi}{\partial x} = 0 \tag{11}$$

where U_c (the area average outflow velocity) is assumed to be unity here and ϕ is a dependent variable, u or v.

The numerical solution of Eqs. (1)–(3) along with the above-noted boundary conditions maps the flow domain $0 \le x \le L_u + L_d$ and $0 \le y \le H$ in terms of the velocity and pressure fields which are further used to obtain the global characteristics like drag and lift coefficient and Strouhal number as well as the values of the stream function and vorticity as follows: