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Research Paper

Development of an implicit material point method for geotechnical applications

Bin Wang [a], Philip J. Vardon [a,*], Michael A. Hicks [a], Zhen Chen [b,c]

[a] Geo-Engineering Section, Faculty of Civil Engineering and Geosciences, Delft University of Technology, P.O. Box 5048, 2600 GA Delft, The Netherlands
[b] Department of Engineering Mechanics, Faculty of Vehicle Engineering and Mechanics, Dalian University of Technology, Dalian 116024, PR China
[c] Department of Civil and Environmental Engineering, University of Missouri, Columbia, MO 65211-2200, USA

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ABSTRACT

An implicit material point method (MPM), a variant of the finite element method (FEM), is presented in this paper. The key feature of MPM is that the spatial discretisation uses a set of material points, which are allowed to move freely through the background mesh. All history-dependent variables are tracked on the material points and these material points are used as integration points similar to the Gaussian points. A mapping and re-mapping algorithm is employed, to allow the state variables and other information to be mapped back and forth between the material points and background mesh nodes during an analysis. In contrast to an explicit time integration scheme utilised by most researchers, an implicit time integration scheme has been utilised here. The advantages of such an approach are twofold: firstly, it addresses the limitation of the time step size inherent in explicit integration schemes, thereby potentially saving significant computational costs for certain types of problems; secondly, it enables an improved algorithm accuracy, which is important for some constitutive behaviours, such as elasto-plasticity. The main purpose of this paper is to provide a unified MPM framework, in which both quasi-static and dynamic analyses can be solved, and to demonstrate the model behaviour. The implementation closely follows standard FEM approaches, where possible, to allow easy conversion of other FEM codes. Newton's method is used to solve the equation of motion for both cases, while the formation of the mass matrix and the required updating of the kinematic variables are unique to the dynamic analysis. Comparisons with an Updated Lagrangian FEM and an explicit MPM code are made with respect to the algorithmic accuracy and time step size in a couple of representative examples, which helps to illustrate the relative performance and advantages of the implicit MPM. A geotechnical application is then considered, illustrating the capabilities of the proposed method when applied in the geotechnical field.

1. Introduction

The material point method (MPM) has been shown to be a robust spatial discretisation method for simulating multi-phase interactions involving large deformations and failure evolution. During 1994–96, Sulsky et al. [1–3] first developed and applied the method for modelling solid materials. This led to researchers, from different fields, recognising the potential of the method and adapting it to various applications, e.g. silo discharge and plastic forming [4,5], explosion problems, exploiting its ability to represent an arbitrary geometry [6,7], large-scale response of cellular constructs in biomechanics [8], and, more recently, for geotechnical analysis, including the modelling of retaining wall failure [4], anchor pull-out [9], soil column collapse [10,11], landslides and debris flows [12], landslide induced interactions with structures [13], and quasi-static analyses of slope stability [14,15].

MPM uses two spatial discretisations, the first one that discretises a continuum body with a set of material points carrying all the state variables, and the second one that discretises the background grid (a computational mesh) to solve the equations of motion. The computational mesh may be maintained in its original position, or it can be adjusted in an appropriate way to avoid mesh distortion after each time/loading step, thereby removing the disadvantage of the finite element method (FEM) for which extreme mesh distortion may occur due to large deformations. As with FEM, the time integration of MPM can be either implicit or explicit, in which the latter has been employed for most of the MPM developments so far. This paper is concerned with the implementation

* Corresponding author.
E-mail addresses: B.Wang@tudelft.nl (B. Wang), P.J.Vardon@tudelft.nl (P.J. Vardon), M.A.Hicks@tudelft.nl (M.A. Hicks), chenzh@missouri.edu (Z. Chen).

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of an implicit MPM framework, and the validation of the resulting implicit solver, as well as with comparisons with an explicit MPM with respect to the time step size and accuracy of the results. In this paper, the term implicit MPM refers to a framework where both dynamic and quasi-static problems (with inertial terms neglected) can be solved effectively. Although implicit dynamic MPM formulations [16,17] have been reported, this paper aims to provide a clear and straightforward description of all the necessary techniques for adapting an existing FEM implementation into one based on the implicit MPM.

In the remaining sections of the paper, the theoretical formulation is first presented. This closely follows the standard FEM procedure, thereby clearly demonstrating the similarities between MPM and FEM. Implementation details are then discussed, where a special treatment for MPM is needed. The subsequent section focuses on a series of representative examples to investigate and validate the presented framework for quasi-static and dynamic analyses, respectively, with the results being compared with those obtained from an explicit code, in order to gain a thorough understanding of how the implicit algorithm behaves.

2. Theoretical formulation of the implicit material point method

2.1. General framework

To describe the implicit MPM, Fig. 1 demonstrates the standard mapping and remapping procedure between the material points and background computational mesh. In the first phase (Fig. 1(a)) the state variables are mapped from the material points to the nodes of the background mesh; in the second phase (Fig. 1(b)), the equation of motion is solved over the background mesh to find the current acceleration, with the element integration being based on the material points (rather than on the information mapped to Gauss points); and, in the third phase (Fig. 1(c)), the state variables on the material points are updated via remapping from the deformed background mesh, and the mesh is then reset, leaving the material points at their updated locations. These phases are repeated until the end of the time/loading steps.

Connectivity can be set up between the material points and background grid nodes, and thus information can be mapped back and forth between them. Due to the different ways that may be adopted for solving the equation of motion in time in the second phase, the formulation can yield either implicit or explicit MPM approaches.

2.2. Continuum equations

At the continuum scale, the governing differential equations under purely mechanical loading can be derived from the respective conservation equations for mass and momentum,

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{v} = 0 \tag{1}$$

$$\rho\frac{d\mathbf{v}}{dt} = \nabla\cdot\boldsymbol{\sigma} + \mathbf{b} \tag{2}$$

supplemented with a suitable constitutive equation to describe the stress–strain relation. In Eqs. (1) and (2), ρ is the mass density, $\mathbf{v}$ is the velocity, $\boldsymbol{\sigma}$ is the Cauchy stress, and $\mathbf{b}$ is the body force due to, for example, gravity.

The mass of a given material point is independent of time, and hence Eq. (1) is automatically satisfied. For Eq. (2), a derivation based on the static equilibrium between the internal force, represented by $\boldsymbol{\sigma}$, and external force, represented by $\mathbf{b}$, is introduced first for simplicity, i.e.

$$\nabla\cdot\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \tag{3}$$

By applying the principle of virtual displacement, followed by the use of the divergence theorem, the equilibrium equation expressed in the weak form [18] with respect to the current configuration, at time $t+\Delta t$, is given by

$$\int_{V^{t+\Delta t}} \mathbf{S}^{t+\Delta t}\cdot\delta\boldsymbol{\varepsilon}^{t+\Delta t}dV = \int_{V^{t+\Delta t}} \mathbf{b}^{t+\Delta t}\cdot\delta\mathbf{u}^{t+\Delta t}dV + \int_{S^{t+\Delta t}} \boldsymbol{\tau}^{t+\Delta t}\cdot\delta\mathbf{u}^{t+\Delta t}dS \tag{4}$$

where $\mathbf{S}$ is the second Piola–Kirchhoff stress tensor, $\delta\boldsymbol{\varepsilon}$ is the Green–Lagrange strain tensor, $\delta\mathbf{u}$ represents the virtual displacement, $\boldsymbol{\tau}$ denotes the prescribed part of the traction on the surface S and the volume of the body is represented by V.

Using the last known configuration at time t as a reference, the stress can be decomposed into the incremental form,

$$\mathbf{S}^{t+\Delta t} = \mathbf{S}^{t} + \Delta\boldsymbol{\sigma} = \boldsymbol{\sigma}^{t} + \Delta\boldsymbol{\sigma} \tag{5}$$

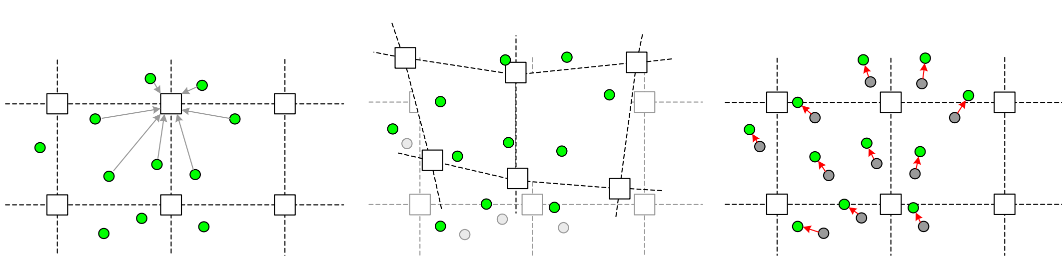
whereas the strain at time $t+\Delta t$, with respect to the time t, is actually the incremental strain $\boldsymbol{\varepsilon}^{t+\Delta t} = \Delta\boldsymbol{\varepsilon}$. The incremental strain is then divided into two parts; a linear part as commonly used in small strain analysis, plus a high order term, i.e. $\Delta\boldsymbol{\varepsilon} = \Delta\mathbf{e} + \Delta\boldsymbol{\eta}$, in which

$$\Delta\mathbf{e} = \frac{1}{2}(\nabla\bar{\mathbf{u}} + (\nabla\bar{\mathbf{u}})^{\mathrm{T}});\quad \Delta\boldsymbol{\eta} = \frac{1}{2}((\nabla\bar{\mathbf{u}})^{\mathrm{T}}\cdot\nabla\bar{\mathbf{u}}) \tag{6}$$

where $\bar{\mathbf{u}}$ is the incremental displacement.

By substituting Eqs. (5) and (6) into the weak formulation (4) and assuming, for the moment, that the loading is deformation independent, then, by expressing the right hand side of Eq. (4) as $\mathbf{F}_{ext}^{t+\Delta t}$, which is the external loading accounting for the effects of both body loads and tractions, and neglecting the high order term $\int_{V^t}\Delta\boldsymbol{\sigma}\cdot\delta\Delta\boldsymbol{\eta}$, the small strain equation of motion in the Updated Lagrangian (UL) formulation is obtained as,

$$\int_{V^t}\Delta\boldsymbol{\sigma}\cdot\delta\Delta\mathbf{e}\;dV + \int_{V^t}\boldsymbol{\sigma}^{t}\cdot\delta\Delta\boldsymbol{\eta}\;dV = \mathbf{F}_{ext}^{t+\Delta t} - \int_{V^t}\boldsymbol{\sigma}^{t}\cdot\delta\Delta\mathbf{e}\;dV \tag{7}$$



(a) Mapping phase (b) UL phase (c) Convection phase

Fig. 1. Computational cycle of MPM (after Sulsky and Schreyer [3]).