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Research Paper

Numerical modelling of multiphase flow in unsaturated deforming porous media

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ABSTRACT

The aim of this paper is to address a number of significant challenges in the analysis of multiphase unsaturated soils when subjected to both static and dynamic loading. These challenges include the non-linear behaviour of the solid skeleton of the soil as well as the means by which the unsaturated nature of the multi-phase soil is dealt with. A review of some fundamental issues in partially saturated soils as well as the governing equations are presented and then the application of the generalised-α algorithm for time integration of the global equations of motion for unsaturated soils is demonstrated. Solutions to these equations obtained by the finite element method are validated by recently presented analytical solutions. A description of the selected constitutive model and its integration is also presented, together with a strategy to verify the numerical implementation. Finally, solutions for the classic problem of static loading of a rigid footing resting on a partially saturated (three-phase) soil and a fully saturated (two-phase) soil are presented.

1. Introduction

The governing equations of solid and fluid interaction were first developed for quasi-static situations by Biot [4], who later extended his analysis to dynamic problems [5]. The subsequent introduction of 'mixture' theory by Truesdell [77] and improvements to this theory by researchers, such as Barden et al. [3], Bowen [9], Green and Naghdi [24], paved the way for the establishment of a new basis for the Thermo-Hydro-Mechanical–Chemical analysis of porous media, allowing for the existence of multiphase pore fluids. This made it possible to incorporate some advanced features of multi-phase material response, including phase changes, chemical reactions, and behaviour under a non-isothermal environment, into the analysis of porous media.

The extension of Biot's theory to include the numerical analysis of saturated soils under dynamic loading was presented by Zienkiewicz and Shiomi [88], and later extensions for partially saturated soils were presented by Li et al. [43] and Li and Zienkiewicz [45]. In these extensions it was assumed that no phase transfer and no chemical reactions were possible during the fluid flows. This kind of flow through porous media is called "immiscible flow", and is the subject of this paper.

In recent decades numerical modelling of multiphase flow in deforming porous media has received increasing attention in various areas of engineering, such as biomechanics and geotechnics. The so-called mixture theory has mostly been employed as the main framework for analysing problems involving thermo-hydro-mechanical and even chemical coupling in porous media. Partially saturated soils can be viewed as a subclass of this type of problem, since they are characterised by the simultaneous deformation of a porous soil, and its pore water and pore air. However, partially saturated soils usually exhibit more complex behaviour than some other porous materials, as they often experience non-linear plastic (irrecoverable) deformation. This makes their computational modelling more complicated than for some other coupled problems. The selection of a consistent constitutive model within the theory of mixtures, that can incorporate suction forces into the description of stress, provides a further complication. The necessity of such incorporation has frequently been reported in experimental studies of unsaturated soils. This requires the adoption of a unique strategy for integration of the constitutive model for these soils. Another challenge in modelling unsaturated soil as a three-phase material arises in solving the global equations of motion, where the presence of both non-saturated and saturated phases, together with the existence of inertia forces in each phase, makes the solution of the coupled dynamic system computationally demanding.

The aim of this paper is to address the aforementioned challenges. A review of some fundamental issues in partially saturated soils is presented and then the application of the generalised-α

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algorithm for time integration of the global equations of motion for unsaturated soils is demonstrated. Solutions to these equations obtained by the finite element method are validated by recently presented analytical solutions. A description of the selected constitutive model and its integration is also presented, together with a strategy to verify the numerical implementation. Finally, solutions for the classic problem of static loading of a rigid footing resting on a partially saturated (three-phase) soil and a fully saturated (two-phase) soil are presented.

2. Governing equations

2.1. Volume fraction and effective density

It is assumed that the medium under consideration is composed of three phases, including solid (s), liquid (w), and gas (g) phases. These phases are continuously distributed throughout space. The degree of saturation, S, and the partial density of each phase ρ are obtained from

$$S_\beta = \frac{V^\beta}{V^v} \quad (\beta = w, g) \tag{1}$$

$$\rho^\alpha = \frac{M^\alpha}{V} \quad (\alpha = s, w, g) \tag{2}$$

where V represent the volume and M is the mass. This gives the average density of the mixture, ρ, as:

$$\rho = (1-n)\rho_s + nS_w\rho_w + nS_g\rho_g \tag{3}$$

where n is the porosity.

2.2. Average effective stress concept in unsaturated soils

During the last seven decades, the concept of 'effective stress' has remained an extremely debatable topic in the analysis of fluid-saturated porous materials. It is also considered as perhaps the first contributor to the study of such materials due to its unique capability to extend the available constitutive models for saturated soils to partially fluid-filled geomaterials, e.g., Khalili et al. [33], Sheng [67,68], Vlahinić et al. [79]. Several researchers have independently published similar definitions of the effective stress for partially saturated materials, such as Alonso et al. [2], Hassanizadeh and Gray [25], Jennings [29], Khalili and Khabbaz [35], Lambe [38], Richards [59].

The single effective stress method emerged following the proposition by Bishop [6] and is similar to that adopted for fully saturated conditions. Bishop proposed the following equation for the effective stress in partially saturated soils:

$$\sigma'_{ij} = (\sigma_{ij} - p_a\delta_{ij}) + \chi(p_a - p_w)\delta_{ij} \tag{4}$$

where σ'_{ij} is the effective stress, σ_{ij} represents the total stress, p_a denotes the pore air pressure, p_w is the pore water pressure, χ is called the effective stress parameter or Bishop's parameter, ranging from 0 to 1 for dry and saturated conditions, respectively, and δ_{ij} is the Kronecker delta. The term $(\sigma_{ij} - p_a\delta_{ij})$ in Eq. (4) is commonly called the net stress, and $p_a - p_w$ represents the matric suction, also known as the capillary pressure.

A popular form of Eq. (4) that was introduced by Lewis and Schrefler [40] is achieved by assuming that the Bishop parameter, χ is identical to the degree of saturation, S_w, so that

$$\sigma'_{ij} = (\sigma_{ij} - p_g\delta_{ij}) + S_w(p_g - p_w)\delta_{ij} \tag{5}$$

A review on the concept of effective stress and the proposed methods of for defining it for unsaturated soils are also presented in Khalili et al. [33]. The application of an alternative effective stress and its role in global system of equations of unsaturated soils is also shown in the work of Khalili et al. [34]. Moreover, investigation on Biot's coefficient in partially saturated media and unsaturated stress tensors by using thermodynamically constrained averaging theory is also presented in the works of Gray and Schrefler [21], Gray et al. [22,23].

Houlsby [27] described σ'_{ij} defined in Eq. (5) as the "average soil skeleton stress" tensor. This representation of effective stress can also be found in Bolzon et al. [7], Ehlers et al. [15], Hutter et al. [28], Jommi [30], Lewis and Schrefler [41], Nuth and Laloui [56], Oka et al. [57], Tamagnini [75], Wheeler et al. [80], with different names adopted, such as "skeleton stress". This definition is consistent with multiphase mixture theory.

In this paper Eq. (5) is selected to define the average effective σ'_{ij} in the mechanics of partially saturated porous media. By also incorporating the Biot parameter, α, Eq. (5) becomes:

$$\sigma'_{ij} = \sigma_{ij} - \alpha \cdot p_{ave} \cdot \delta_{ij} \tag{6}$$

where p_{ave} denotes the mean pore pressure exerted by the fluid phase(s) on the solid grains, and is obtained by the averaging technique proposed by Gray and Hassanizadeh [20], Houlsby [27], according to:

$$p_{ave} = S_w \cdot p_w + S_g \cdot p_g \tag{7}$$

Substituting the capillary pressure, p_c, into Eq. (7) gives:

$$p_{ave} = p_w + (1 - S_w) \cdot p_c \tag{8}$$

The Biot coefficient, α, is described by the relationship:

$$\alpha = 1 - \frac{K_t}{K_s} \leqslant 1 \tag{9}$$

where K_t is the bulk modulus of the porous medium and K_s is the bulk modulus of the solid grains.

2.3. Conservation of mass and momentum balance

According to the principle of conservation of mass, inside an arbitrary volume V, mass cannot be eliminated or increased unless there is an outward or inward flow of materials, respectively. A flow of each phase $(\alpha = s, w, g)$ through a surface $d\Gamma$ can be described as $\rho_\alpha n_\alpha \cdot \mathbf{V}^\alpha \cdot \mathbf{n} \cdot d\Gamma$, where $\mathbf{n}$ is the normal unit vector of the surface, ρ_α is the partial mass density of each material, and $\mathbf{V}^\alpha$ is the velocity of the material. Also $n_\alpha = \frac{\Omega^\alpha}{\Omega}$ represents the volume fraction of each phase in the mixture with Ω^α and Ω as occupied volume by each phase and total volume of the mixture, respectively.

In its integral form, the principle of mass conservation is written as

$$\frac{\partial}{dt}\int_\Omega \rho_\alpha n_\alpha d\Omega = -\oint_\Gamma \rho_\alpha n_\alpha \cdot \mathbf{V}^\alpha \cdot \mathbf{n} \cdot d\Gamma \tag{10}$$

By applying the Gauss theorem, the conservation of mass can be written in the form of a differential equation as follows

$$\frac{\partial(\rho_\alpha n_\alpha)}{\partial t} + \frac{\partial(\rho_\alpha n_\alpha V_i^\alpha)}{\partial x_i} = 0 \tag{11}$$

Considering $\mathbf{V}^s = \dot{\mathbf{u}}$ for solid phase and porosity as $n = n_w + n_g$, the following expression for the conservation of mass for solid phase can be obtained as:

$$\frac{\partial((1-n)\rho_s)}{\partial t} + \frac{\partial((1-n)\rho_s \dot{u}_i)}{\partial x_i} = \mathbf{0} \tag{12}$$

Assuming $\frac{\partial(\Re_i)}{\partial x_i} = \Re_{i,i}$ and $\frac{\partial \rho_s}{\partial x_i} = 0$ the following equation is obtained from expanding the equation above: