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# Study on the pulsating flow of a worm-like micellar solution

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ABSTRACT

In this work, the rectilinear flow of a complex liquid under a pulsating, time-dependent pressure gradient is analyzed. The fluctuating component of the pressure gradient is assumed to be of small amplitude and can be adequately represented by a weakly stochastic process, for which a quasi-static perturbation solution scheme is suggested. The pulsating pressure-gradient flow is analyzed with the Bautista–Manero–Puig model (BMP) constitutive equation, consisting in the upper convected Maxwell equation coupled to a kinetic equation to account for the breakdown and reformation of the fluid structure. According to the BMP model, thixotropy was found to have a negative effect on the energy associated to the maximum flow enhancement and reflects the relationship among the kinetic, viscous and structural mechanisms in the system. The flow enhancement is a function of the square of the amplitude of the oscillations, the Reynolds and Weissenberg numbers, and it is also dependent on the dimensionless numbers representing the viscoelastic, kinetic and structural mechanisms. Finally, flow enhancement is predicted in an aqueous worm-like micellar solution of cetyltrimethyl ammonium tosilate (CTAT) for various concentrations.



## 1. Introduction

The analysis of the oscillating pressure gradient flow of Newtonian and non-Newtonian fluids has attracted ample interest due to several applications, among them, in bio-fluid mechanics [1–3], biorheology, enhanced oil recovery and others. In biorheology, examples are the flow of blood in veins which is forced by a periodic pressure gradient [4–10] and interesting manifestations of biological fluid flow such as the flow of spider silk [11–13]. From a practical point of view, pulsatile flow of complex liquids (worm-like micellar systems and lyotropic liquid crystals) has applications in enhanced oil recovery. Likewise, pulsating and oscillating flows are important in the industrial applications such as polymer extrusion using oscillatory dies. The effect of the oscillations on the heat transfer and their interplay with inertia and viscous dissipation in non-Newtonian fluids, such as the dependency of the bulk temperature on frequency and amplitude of the oscillations, has been reported [14–21]. In addition, the use of pulsations has also been of interest in connection with heat, turbulent heat, mass transfer and coating processes [22–24].

Constitutive equations that take into account build-up and break-down kinetics of a complex fluid structure have been used to model several complex systems [25]. Complex fluids include biological fluids such as polypeptides, cellulose, and composites, that often exhibit crystalline order, anisotropy and viscoelasticity. They incorporate sequences of self-assembly, structural, kinetic processes under flow and mass transfer [26]. The characterization of these phenomena has been studied at length by several authors [27–31].

Likewise, viscoelastic surfactants have been used as rheological modifiers in coating process and also in enhanced oil recovery operations, especially those related to underground formations. The extraction of additional amounts of oil can be achieved by hydraulically inducting fractures in the rock formations [32]. Viscoelastic surfactants are characterized by entangled network of large worm-like micelle structures. These structures break and reform during flow, exhibiting a rich rheological behavior. Predictions of the flow behavior of viscoelastic surfactants by constitutive equations have been a challenging issue [33,34]. These systems exhibit Maxwell type behavior in small-amplitude oscillatory shear flow and saturation of the shear stress in steady simple shear, which leads to thixotropy and shear banding flow [35–37]. In the non-linear viscoelastic regime, elongated micellar solutions also exhibit remarkable features, such as the presence of a stress plateau in steady shear flow past a critical shear rate accompanied by slow transients to reach steady state [38,39].

Theoretical predictions using perturbation and numerical methods on viscometric flows (or nearly viscometric flows) of the flow enhancement as a function of frequency and amplitude of oscillations have been reported [39–71], using viscous and viscoelastic

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equations of state [39–41,49,50,54,55,57–60,67–71]. In most analyses, it is shown that shear-thinning causes the flow enhancement and that this enhancement is proportional to the square of the relative amplitude of the oscillating pressure gradient and its magnitude depends strongly on the shape of the viscosity function. The maximum in the resonance curves reported by several authors can be explained by a coupling of the viscoelastic properties with the macroscopic perturbed motion. Among the important quantities are the shape of the viscosity curve and the inter-relation of the characteristic material properties of the system [54–60]. Other important factors are the wave-form (triangular, sinusoidal, and square type) that has a strong effect on flow enhancement and power requirements [64,65,68–70].

Notwithstanding, there is still open questions and lack of theoretical and experimental studies dealing with complex fluids and complex behaviors such as thixotropy, rheopexy and shear-banding in pulsating and oscillating flows. They represent a test to new constitutive equations and this aspect motivates the present investigation. In this regard, the main objectives of this work are:

1. Predictions of the flow enhancement and power requirement by a pulsating time pressure gradient of a complex kinetic liquid modelled by the Bautista–Manero–Puig (BMP) equation of state [72–75].
2. Analysis of the thixotropy and the inter-play with the kinetic, structural and viscoelastic properties of the fluid, through dimensionless groups associated to each mechanism.
3. Study the effect of the surfactant concentration on thixotropy using rheometric data of an aqueous worm-like micellar solution (cetyl trimethyl ammonium tosilate) to predict the flow enhancement for various micellar concentrations [76].

This paper is organized as follows: Section 1 contains the introduction to the problem and previous work. Section 2 discusses the BMP model. The formulation to the problem is presented in Section 3, with the non-dimensional variables and the stochastic properties of the random function $n(t)$ used to describe the pulsating pressure gradient. In Section 4, the perturbation solution is proposed and analytical results are shown in Section 5. Theoretical predictions of the flow enhancement using worm-like solutions data are described in Section 6. Concluding remarks and future work are mentioned in the last two sections.

## 2. Constitutive equation (the BMP model)

The BMP model [72–75] couples a time dependent equation for the structure changes with the upper-convected Maxwell constitutive equation. The evolution equation for the structural changes was conceived to account for the kinetic process of breakage and reformation of complex liquid and is defined by the following set of equations:

$$\underline{\underline{\sigma}} + \frac{\eta(II_{\underline{\underline{D}}})}{G_0}\overset{\nabla}{\underline{\underline{\sigma}}} = 2\eta(II_{\underline{\underline{D}}})\underline{\underline{D}} \tag{1}$$

$$\frac{d}{dt}\ln\ \eta(II_{\underline{\underline{D}}})^{\lambda} = 1 - \frac{\eta(II_{\underline{\underline{D}}})}{\eta_0} + k\lambda\left(1 - \frac{\eta(II_{\underline{\underline{D}}})}{\eta_\infty}\right)\underline{\underline{\sigma}} : \underline{\underline{D}} \tag{2}$$

In Eq. (1) the stress $\underline{\underline{\sigma}}$ is a viscoelastic stress, $\overset{\nabla}{\underline{\underline{\sigma}}}$ is the upper-convected derivative of the stress tensor, $\eta$ is the viscosity function, $\underline{\underline{D}}$ is the rate of deformation tensor, $II_{\underline{\underline{D}}}$ is the second invariant of $\underline{\underline{D}}$ and $G_0$ is the elastic modulus. In Eq. (2) $\eta_0$ and $\eta_\infty$ are the viscosities at zero and very high shear rates, respectively, $\lambda$ is the structural relaxation time and $k$ can be interpreted as a kinetic constant for the structure breakdown; all five parameters of the model ($\eta_0$, $\eta_\infty$, $G_0$, $\lambda$, and $k$) are related to the fluid properties and can be estimated from independent rheological experiments in steady and unsteady flows. The viscosity at lower and upper shear rates ($\eta_0$, $\eta_\infty$) can be estimated through experiments in steady shear flow. The structural time and elastic modulus ($G_0$, $\lambda$) can be calculated by using linear oscillatory flow. The parameter ($k$) can be evaluated in stress relaxation experiments after steady shear flow [76,77].

The BMP model was selected for this study due to its ability to predict the thixotropic behavior of structured fluids (such as worm-like micellar solutions, dispersions of lamellar liquid crystals, bentonite suspensions and associative polymers) [72–78]. It reproduces the flow curve of shear-thinning fluids, i.e. a Newtonian plateau at low and high shear rates and the intermediate power law region. Due to its simplicity, analytical solutions for complex flow situations can be explored, as compared to other more complex models [79–84].

## 3. Problem formulation

The isothermal rectilinear flow of an incompressible complex liquid under a pulsating time-dependent pressure gradient is analyzed in a circular pipe of radius $r = a$ and axial length $z = L$. Entry and exit effects and gravitational forces are neglected. In this system, all physical quantities in cylindrical coordinates $(r, \theta, z)$ are defined with respect to an origin at the pipe center. The axial fluid velocity is a function of $(r, t)$ and both the non-slip condition $(V_z(r = a, t) = 0)$ and symmetry of the velocity field are applied. The pulsating pressure gradient here is represented by $\partial_z p(1 + \varepsilon n(t))$, where $n(t)$ is a pressure gradient noise and $\varepsilon \ll 1$ is a small parameter.

### 3.1. Dimensionless variables, groups and equations

#### 3.1.1. Non-dimensional variables

Herrera et al. [71,85] proposed the following dimensionless variables for the axial velocity, pressure gradient, time, shear-stress, shear-rate, radial coordinate, viscosity function and frequency, respectively

$$V_z^* = \frac{V_z}{\omega a}; \quad p = \frac{dP/dz}{\eta_0/a\lambda}; \quad t^* = \frac{t}{\lambda}; \quad \tau = \frac{\sigma_{rz}}{\eta_0/\lambda};$$
$$N_{(1)}^* = \frac{N_1}{\eta_0/\lambda} = \frac{\sigma_{rr} - \sigma_{\theta\theta}}{\eta_0/\lambda}$$
$$N_{(2)}^* = \frac{N_2}{\eta_0/\lambda} = \frac{\sigma_{\theta\theta} - \sigma_{zz}}{\eta_0/\lambda}; \quad \dot{\gamma}^* = \lambda\dot{\gamma}_{rz};$$
$$r^* = \frac{r}{a}; \quad \eta^* = \frac{\eta}{\eta_0}; \quad \omega^* = \omega\lambda \tag{3}$$

Here, the characteristic time is $\lambda$ (structural build-up time). This election of the non-dimensional variables enables the comparison with other characteristic times associated to a given physical mechanism (e.g. viscoelastic, $\lambda_0 = \eta_0 G_0^{-1}$, $\lambda_\infty = \eta_\infty G_0^{-1}$ and rupture $\lambda_r = k\eta_0$ times).

#### 3.1.2. Non-dimensional groups

Using the above expressions, the dimensionless components of the momentum equation, constitutive equations and the flow enhancement are obtained. In addition, the following non-dimensional groups are defined, as discussed previously by Herrera et al. [71,85]