Computers and Geotechnics 69 (2015) 114–123

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Analytical solutions for contaminant transport in a semi-infinite porous medium using the source function method

Bing Bai *, Huawei Li, Tao Xu, Xingxin Chen

School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, PR China

A R T I C L E I N F O

Article history:
Received 5 December 2014
Received in revised form 5 April 2015
Accepted 6 May 2015

Keywords:
Hydraulic permeability
Contaminant transport
Release effect
Dispersive coefficient
Elementary solution

A B S T R A C T

The general solutions for contaminant transport in a saturated semi-infinite porous media are derived by using the Laplace transform and Fourier transform, along with their transform inversions, under the conditions of one-dimensional seepage flow and the three-dimensional dispersive effect. The analytical expression of contaminant concentration in a porous medium, subjected to a local contaminant source with arbitrary geometry, and intensity that varies with time and coordinates is derived by the source function method based on the elementary solution of an instantaneous point contaminant source. The results show that an exponentially degenerated contaminant source injected into the porous medium migrates gradually toward the depth and width of the porous medium due to the convective water flow and diffusion induced by molecular and mechanical movement, along with deposition of the contaminant on the solid matrix surface. The contaminant concentration in a porous medium subjected to a cyclic contaminant source exhibits cyclic fluctuations due to the fluctuation of the contaminant source applied on the porous surface; concentrations reach a quasi-steady state, with the same fluctuation phase as the contaminant source. The hydrodynamic dispersion effect accelerates the migration processes of the contaminant in the vertical direction as well as the diffusion in the horizontal direction, resulting in a dramatic rise in the contaminant concentration in a short period of time.

1. Introduction

Studies of contaminant transport in porous media based on hydraulic permeability are of great importance for a wide range of engineering, biophysical, and biomedical applications, such as contaminant treatment, oil extraction, and disposal of high-level nuclear waste [1,2]. Over the past few decades, intense research has been conducted to better understand the migration process, transport mechanism, and absorption and release effects of contaminants, such as lead, arsenic, mercury, cadmium, colloid particles, phosphorus, and plant nitrogen nutrients [3–5].

Several mathematical models have been developed to account for contaminant transport in porous media under different conditions of groundwater movement, with particular attention paid to the migration mechanism of pollutants. Chang et al. [4] simulated copper and cadmium transport in a lateritic silty-clay soil column by using the Freundlich nonlinear equilibrium-controlled sorption parameters to determine the retardation factor used in column leaching experiments. Jungnickel et al. [5] described the solution of a fully coupled set of transport equations describing the simultaneous diffusion of several ion species through a clayey soil. Seetharam et al. [6] derived a multicomponent reactive transport model, coupled with an existing thermal, hydraulic, and mechanical model for porous media, based on conservation of mass/energy principles for flow and stress–strain equilibrium. Considerable effort has also been devoted to the modeling of consolidation-induced contaminant transport. Peters and Smith [7] derived flow and transport equations for a deforming porous medium based on the mass balance law and illustrated the differences between the theory for the rigid porous medium and deforming porous medium using solute transport through an engineered landfill liner as an example. Fox et al. [8] experimentally and computationally investigated the importance of the consolidation process to solute transport in compressible porous media.

Analytical solutions for more complex problems, such as variable coefficients, unsteady flow, and multi-layered media, have gained attention in recent years. Kumar et al. [9] used the Laplace transform to derive the analytical solutions for the one-dimensional advection–diffusion equation, with variable

* Corresponding author. Tel.: +86 010 51684815.
E-mail address: Baibing66@263.net (B. Bai).

http://dx.doi.org/10.1016/j.compgeo.2015.05.002
0266-352X/

Nomenclature

List of symbols

x, y, z	spatial coordinates
C	concentration of the liquid-phase solutes
n	porosity of the porous medium
D_x, D_y, D_z, D_r	hydrodynamic dispersion coefficients
u	average interstitial velocity
t	time
σ	concentration of the solutes deposited onto the solid matrix
ρ_s	bulk density of the solid matrix
k_d	decay coefficient of the liquid-phase solutes
k_r	decay coefficient of the sorbed solutes
s, r	Laplace transform variables of t and z
ω, ϕ	Fourier transform variables of x and y
L^{-1}	Laplace inverse operator
α	arbitrary constant
I_0, I_1	modified Bessel function of the first kind of order zero and one
α_1, α_2	arbitrary constants
p, l, τ	dummy integration variables
I	strength of the point source
M	quantity of contaminant injected
$\delta(\cdot)$	Dirac delta function
x', y'	coordinates on the contaminant source plane
t'	time of the contaminant injection
a	radius of the contaminant source
r, φ	cylindrical coordinates
q_i	initial concentration of the contaminant source
β	degradation parameter
k	parameter describing the geometric shape of the contaminant source
z_0	influence depth of the contaminant
q_a, q_0	average intensity and half-amplitude of the cyclic contaminant source
ω, Z	frequency and period of the cyclic contaminant source

coefficients and initial and boundary conditions in a semi-infinite domain for three dispersion problems; the solutions could be used for a variety of dispersion and velocity variations due to their spatial and temporal dependence. Li and Cleall [10] presented analytical solutions for conservative solute diffusion in one-dimensional double-layered porous media; these solutions are applicable to various combinations of fixed solute concentrations and zero-flux boundary conditions applied at each end of a finite one-dimensional domain and can consider arbitrary initial solute concentration distributions throughout the media. Guerrero and Skaggs [11] presented a general analytical solution for a linear, one-dimensional advection–dispersion equation with distance-dependent coefficients, deduced a transport equation with a self-adjoint differential operator by using an integrating factor, and obtained the solution using the generalized integral transform technique. Yadav et al. [12] discussed the analytical solution of a one-dimensional linear advection–diffusion equation by using the Laplace integral transform, in which the solute transport and flow field were considered to be unsteady independent patterns. Chen et al. [13] presented exact analytical solutions to the two-dimensional advection–dispersion equation in cylindrical coordinates in a finite domain subject to the first-type and third-type inlet boundary conditions (i.e., the Dirichlet boundary condition and mixed boundary condition) using the finite Hankel transform of the second kind and the generalized integral transform technique.

However, previous studies have largely been limited to the classical advection–dispersion governing equation on the principle of conservation of mass and Fick's laws of diffusion, as well as traditional contaminant injection models, such as first- or third-type boundary conditions. In the present study, the governing equations of contaminant transport in a saturated porous medium with the release effect of sorbed solutes were established to account for the one-dimensional flow and three-dimensional dispersive effect. The elementary solution for the case of a point contaminant source instantaneously applied on the surface of a porous medium was derived by using the Laplace and Fourier transforms, along with their transform inversions. The integration method is then used to develop analytical expressions for contaminant concentrations in a porous medium subjected to a local contaminant source with arbitrary geometry and intensity varying with time and coordinates on its free surface.

2. Governing equations and solutions

2.1. Governing equations

This analysis assumes that the contaminants are injected from the local surface of a half-space, saturated, homogeneous porous medium (i.e., infinite in the x and y directions but semi-infinite in the z direction). The transport process of solutes during one-dimensional steady-state flow in the z direction, accounting for three-dimensional hydrodynamic dispersion, is governed by the following partial differential equation [14,15]:

$$\frac{\partial C(x,y,z,t)}{\partial t} = D_x \frac{\partial^2 C(x,y,z,t)}{\partial x^2} + D_y \frac{\partial^2 C(x,y,z,t)}{\partial y^2} + D_z \frac{\partial^2 C(x,y,z,t)}{\partial z^2} - u\frac{\partial C(x,y,z,t)}{\partial z} - \frac{\rho_s}{n}\frac{\partial \sigma(x,y,z,t)}{\partial t} \tag{1}$$

where x and y are the spatial coordinates perpendicular to the water flow, z is the coordinate parallel to the water flow, C is the concentration of the liquid-phase solute, n is the porosity of the medium, D_x, D_y and D_z are the hydrodynamic dispersion coefficients in the x, y and z directions, respectively, u is the average interstitial velocity, t is the time, σ is the concentration of solute deposited onto the solid matrix, and ρ_s is the bulk density of the solid matrix.

The first three terms on the right-hand side of Eq. (1) represent the hydrodynamic dispersive effect in the x, y and $z$directions; the fourth term represents the convective effect caused by hydraulic permeability; and the fifth term represents the dynamic deposition of contaminants accounting for the release effect, which can be written as [14,16]

$$\frac{\rho_s}{n}\cdot\frac{\partial \sigma(x,y,z,t)}{\partial t} = k_d\cdot C(x,y,z,t) - k_r\cdot\frac{\rho_s}{n}\cdot\sigma(x,y,z,t) \tag{2}$$

where k_d and k_r are the decay coefficients of the liquid-phase and sorbed solutes, respectively.

2.2. Solutions in the transform space

Assuming that there are initially no sorbed solutes present in the porous formation, the solutes are injected from the surface of a half-space in concentrations varying with time and coordinates. The initial and boundary conditions are set as follows: