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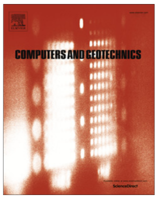

Research Paper

# 3D zero-thickness coupled interface finite element: Formulation and application

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ABSTRACT

In many fields of geotechnical engineering, the modelling of interfaces requires special numerical tools. This paper presents the formulation of a 3D fully coupled hydro-mechanical finite element of interface. The element belongs to the zero-thickness family and the contact constraint is enforced by the penalty method. Fluid flow is discretised through a three-node scheme, discretising the inner flow by additional nodes. The element is able to reproduce the contact/loss of contact between two solids as well as shearing/sliding of the interface. Fluid flow through and across the interface can be modelled. Opening of a gap within the interface influences the longitudinal transmissivity as well as the storage of water inside the interface. Moreover the computation of an effective pressure within the interface, according to the Terzaghi's principle creates an additional hydro-mechanical coupling. The uplifting simulation of a suction caisson embedded in a soil layer illustrates the main features of the element. Friction is progressively mobilised along the shaft of the caisson and sliding finally takes place. A gap is created below the top of the caisson and filled with water. It illustrates the storage capacity within the interface and the transversal flow. Longitudinal fluid flow is highlighted between the shaft of the caisson and the soil. The fluid flow depends on the opening of the gap and is related to the cubic law.

## 1. Introduction

The role of interfaces and discontinuities is crucial in many fields of geotechnical engineering and engineering geology. They cover a wide range of scales from soil-structure interaction to geological faults. In all cases, the interface delineates two distinct media and has a very thin width with respect to them. They often constitute preferential paths for fluid flows, deformation and failure. Therefore the modelling of their behaviour is a major issue for engineers.

Assessing the behaviour of foundations requires a deep understanding of the interface mechanisms. Prediction of the frictional strength of a pile is crucial to estimate and model its resistance to driving [1–3]. Soil-foundation friction is also a major component of the resistance of anchors or pile foundations to pull loading [4–6]. The modelling of limit states or post-failure behaviours of these foundations requires specific numerical tools able to take into account large relative displacements between the foundation and the surrounding soil.

Suction caissons or bucket foundations are a particular case of anchors. They may be used as permanent foundations for offshore structures [7–9]. They consist of steel cylinders open towards the bottom. They are installed within the soil by suction [10,11], i.e. the water inside the caisson is pumped out creating a fluid flow from outside. This creates a differential of water pressure between inside and outside, digging the caisson into the soil. This suction effect is also mobilised during the loading of the foundation especially in traction [4,12]. It increases the total transient resistance of the foundation. It also ensures the foundation does not fail even after full mobilisation of friction between the soil and the caisson. Correctly representing the large uplifting of the caisson and the mobilisation of friction are among the main challenges of their modelling [13,14].

The behaviour of geological faults in the vicinity of hydrocarbon production wells was given much attention [15,16]. Disturbances created by such a process may affect the environment in triggering micro-earthquakes or inducing settlements. Recently the possibility of carbon dioxide geological storage in reservoirs has given a new impetus to this topic [17]. The fault opening may create a leakage path from the storage, fracture the caprock [18] or trigger earthquakes [19].

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**Nomenclature**

*Roman symbols*

| | |
|---|---|
| $(\mathbf{e}_1^1, \mathbf{e}_2^1, \mathbf{e}_3^1)$ | local system of coordinates defined on mortar side |
| $(\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3)$ | global system of coordinates |
| $f_{wl}$ | longitudinal fluid flux within the interface |
| $f_{wt}$ | transversal fluid flux across the interface |
| $\mathbf{F}_E, \mathbf{F}_I, \mathbf{F}_{OB}$ | external, internal and out of balance energetically equivalent nodal forces |
| $\mathrm{g}_N, \dot{\mathrm{g}}_N$ | gap function, variation of this function |
| $\dot{\mathrm{g}}_T$ | variation of tangential displacement |
| $\mathbf{J}$ | Jacobian of the transformation from actual to isoparametric element |
| $k$ | intrinsic permeability |
| $\mathbf{K}$ | stiffness matrix |
| $\mathrm{K}_N, \mathrm{K}_T$ | penalty coefficients |
| $\mathrm{p}_N, \mathrm{p}'_N$ | contact pressure, effective contact pressure |
| $\mathbf{R}$ | rotation matrix |
| $\dot{\mathrm{S}}$ | storage |
| $\mathbf{t}$ | local contact stress vector |
| $\mathrm{T}_{wt}$ | transversal conductivity |
| $\mathbf{u}$ | vector of generalised coordinates $(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{p}_w)$ |
| W | Gauss weight |

*Greek symbols*

| | |
|---|---|
| $\Gamma_c^1$ | area of contact |
| $\Gamma_{\tilde{q}}^1$ | area of the non-classical fluid boundary condition |
| $\delta\dot{\mathbf{x}}$ | virtual field of velocities |
| $\delta \mathrm{p}_w$ | virtual field of fluid pressures |
| $\boldsymbol{\epsilon}$ | deformation tensor |
| $\mu$ | friction coefficient |
| $\rho_w$ | fluid density |
| $\boldsymbol{\sigma}$ | stress tensor |
| $\tau$ | tangential contact shear stress |
| $\phi^i(\xi, \eta)$ | interpolation function related to node *i*, in the isoparametric system of coordinates |
| $\Omega^i$ | porous medium nb. *i*, solid nb. *i* |

*Mathematical symbols*

| | |
|---|---|
| $\boldsymbol{\nabla}$ | gradient operator |
| $:$ | tensor contraction |
| $\cdot$ | scalar product |
| $[\cdot]^T$ | transpose operator |
| $[\cdot]^{-1}$ | inverse operator |
| $\Vert \cdot \Vert$ | norm |
| $\delta_{ij}$ | Kronecker delta |

From the numerical point of view, the problem of contact between two solids are early developed. The first purely mechanical finite element of contact between two solids was early developed [20]. It allows these solids to get into contact or to loose contact during a simulation. The main concepts of this field are established during the eighties [21–24] and consolidate during the nineties [25–29]. Many authors developed these elements in the mechanical field of research and especially metal forming [30–32].

Rock and soil mechanics largely contribute to constitutive modelling of interfaces [33–35]. The first improvement is the development of non-linear mechanical constitutive laws characterising rock joints or soil-structure interface. Criteria defining the maximum friction available and stress–strain relations are developed in [34,36–38]. A special attention is paid to the characterisation of shear-induced dilatancy [34,39,40]. The second improvement is the definition of experimental relations characterising the fluid flow within the rock joints [41,42]. Coupled finite elements combine these two ingredients. They include hydro-mechanical [43–46] or multi-phase couplings [47]. They take into account the fluid or multiphasic flow across and within the interface and its effect on the normal pressure acting on the joint.

The purpose of this paper is to present a versatile formulation of a fully coupled hydro-mechanical finite element of interface applicable to 3D simulations. It allies a mechanical large displacement formulation of a zero-thickness interface element with the modelling of fluid flow using a three-node strategy. This strategy discretises the field of fluid pressure on each side of the interface and inside it. Thence, the transversal fluid flow creates a drop of pressure across the interface. The element is hydro-mechanically coupled through the definition of an effective contact pressure, the fluid storage due to the gap opening and the variation of the interface longitudinal permeability with gap variation.

The originality lies in the coupling of the longitudinal and transversal flows within the interface to a classical formulation of mechanical contact in large displacements. Particularly this flow problem is also tackled in case of contact loss and large tangential displacements. Moreover both mechanical and flow problems are treated within a unique finite element code `LAGAMINE` developed at the university of Liege [48,49]. This paper focuses on the general framework of the finite element of interface. However the formulation is very versatile and any constitutive law describing both mechanical and flow behaviours can be introduced instead of the proposed ones. An original application to the large uplift simulation of a suction caisson is provided to illustrate the capacities of the finite element of interface.

This paper is subdivided into four main parts. The first part describes the basics of interface finite elements. It explains the different ways to tackle and discretise mechanical contact and fluid flow within interfaces. The second part sets out the governing equations of the coupled problem and its continuum formulation. The third part displays the discretisation of this continuum formulation into finite elements. It consists of the definition of energetically equivalent nodal forces and stiffness matrix. Finally the last part describes the pull simulation of a suction caisson embedded in a soil layer. This application illustrates all the features of the interface element.

## 2. Review of interface finite elements

Coupled interface elements involve two distinct but related issues: the mechanical and the flow problems. The former describes the detection or the loss of contact between two bodies, the shearing of this contact zone… The flow problem describes the fluid flow within the interface created by the vicinity of fluid flows within porous media. These two problems are coupled since the fluid flow influences the opening of the discontinuity and its transmissivity. Moreover the fluid flow across the interface creates a transversal drop of pressure between two porous media.

Numerically, two approaches exist within the framework of the finite element method to manage the mechanical contact between two bodies as shown in Fig. 1. In the former approach, the interface zone is represented by a very thin layer of elements specially designed for large shear deformation [50–52]. The second approach, adopted in the following, involves special boundary elements. These elements have no thickness and are termed *zero-thickness* finite elements. They discretise the probable zone