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Research Paper

Bearing capacity of strip footings on two-layered clay under combined loading

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ABSTRACT

In this paper, a lower bound limit analysis in conjunction with finite elements and second-order cone programming (SOCP) is used to determine the bearing capacity of a rigid strip footing placed on two-layered clay subjected to inclined or eccentric loading. The footing is founded on the free surface of the soil mass with no surcharge applied. Two types of footing–soil interfaces are considered: (1) zero tensile capacity and (2) non-zero tensile capacity. The numerical results are presented in the form of failure envelopes in the loading plane, and the corresponding failure mechanisms are also presented. The size and shape of the failure envelopes are dependent on (1) the undrained shear strength ratio (c_{u1}/c_{u2}), where c_{u1} and c_{u2} are the undrained shear strength of the top and bottom clay, respectively, and (2) the normalized thickness D/B of the upper clay, where D is the thickness of the top clay layer and B is the width of the footing.

1. Introduction

The ultimate capacity of strip footings placed on homogeneous soil can be estimated using the classic Terzaghi equation and its associated bearing capacity factors. However, in reality, soil strength profiles are not homogeneous and may consist of distinct layers with significantly different properties. If the thickness of the upper layer is large compared to the width of the foundation, realistic estimates of the bearing capacity may be obtained from classic bearing capacity theory based on the properties of the upper layer. If the thickness of the upper layer is small or comparable to the foundation width, then the classic theory may not be appropriate [4]. Because the failure mechanism will extend to the underlying layer, the homogeneity assumption in the classic theory fails.

The bearing capacity of strip footings on a horizontally layered soil profile has been investigated using the limit equilibrium method [24,8] and upper-bound limit analysis with a simplified circular failure mechanism [3,5]. Although the upper bound limit analysis may not lead to a conservative prediction of the limit load, it is still popular in the literature because of its simplicity. Its accuracy can be improved through the use of more complex and more realistic failure mechanisms. For example, a rigid-block collapse mechanism was used by Florkiewicz [7], Michalowski and Shi [21], and Michalowski [22], and a continuous deformation mechanism was used by Michalowski and Shi [21] and Michalowski [22]. Semi-empirical approaches have also been proposed based on a series of model footing tests [2]. Numerical methods such as FEM [13,4,30], which can handle layered soil profiles, have also been applied to this problem. Recently, Merifield et al. [19] developed rigorous lower and upper bound solutions to the bearing capacity of two-layered clay based on the method from Sloan [25] and Sloan and Kleeman [26].

However, most of the previous works only consider vertical loads. In practice, foundations can be subjected to horizontal loads and moments, for example, the wind and wave forces in offshore environments. Therefore, the stability of strip footings on layered soils under combined loading is of practical interest. Unfortunately, available work on this problem is very limited in the literature, except for the works of Meyerhof and Hanna [20], Georgiadis and Michalopoulos [8], Youssef-Abdel Massih et al. [29], and Zhan [31]. The solutions of Meyerhof and Hanna [20] for the case of inclined loading were based on a series of model footing tests from which empirical and semi-empirical bearing capacity factors were derived. Georgiadis and Michalopoulos [8] developed a numerical method of slip surfaces (force equilibrium) for layered soils both for eccentric and inclined loads. Youssef-Abdel Massih et al. [29] investigated the bearing capacity of a strip footing resting on a two-layer foundation soil (sand and clay) in the case of inclined and/or eccentric loads using the

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kinematical approach in limit analysis, in which the translational (for the case of an inclined load) and rotational (for the case of an eccentric load) failure mechanisms were used. The general failure mechanism used by Georgiadis and Michalopoulos [8] is rather simple, and the accuracy of the results is questionable. In addition, the solutions of Youssef-Abdel Massih et al. [29] are upper bounds that may be unsafe, whereas the solutions of Meyerhof and Hanna [20] are largely empirical. In summary, numerical analyses of the ultimate bearing capacity of strip footings on layered soils under combined loading can be improved in several aspects identified above, particularly in terms of rigor and generality of the results. In the work of Zhan [31], the bearing capacity of strip footings on two-layered clay under inclined, eccentric and eccentric-oblique loading is investigated using the finite element method, where no sustained tension is assumed for the footing–soil interface. However, the case of interface with tensile capacity is not investigated. This refers to a fully bonded foundation–soil interface particularly relevant to offshore shallow foundations, which are often equipped with a circumferential skirt to achieve embedment. During loading, suctions develop within the soil plug, providing tensile capacity for the duration over which undrained conditions can be sustained [11].

The aim of this paper is to determine the ultimate bearing capacity of strip footings placed on two-layered clay under inclined or eccentric loading using the lower bound limit analysis in conjunction with finite elements and SOCP, as described in Tang et al. [27]. The results are presented in the form of load interaction diagrams in the (H, M, V) space for different values of the thickness of the upper clay layer and soil strength profiles, where H, M and V are the horizontal force, the overturning moment, and the vertical load, respectively. These curves define a region inside which all allowable loading combinations should lie. In other words, they are ultimate limit states that can be applied to design. The velocity fields are also studied to show the failure of a strip footing on two-layered clay subjected to combined loading. Some new expressions for the ultimate vertical capacity and the failure envelopes are proposed to fit the lower bound results of the ultimate capacity of strip footings on two-layered clay. The results presented in this paper are useful for offshore foundations, as several types of offshore foundations are essentially shallow footings (e.g., the spudcan footings of jack-up units, mudmats for fixed jackets, and concrete gravity bases).

2. Problem definition

Fig. 1 shows the case of a strip footing of width B resting on two-layered clay, where the upper layer of clay is characterized by undrained shear strength c_{u1} and thickness D. This layer is underlain by a clay layer of undrained shear strength c_{u2} that extends beyond the influence zone of the foundation (deeper than $10B$). The bearing capacity will be a function of the two ratios D/B and c_{u1}/c_{u2}. In this paper, solutions will be computed for problems where D/B ranges from 0.125 to 2 and c_{u1}/c_{u2} varies from 0.1 to 5.

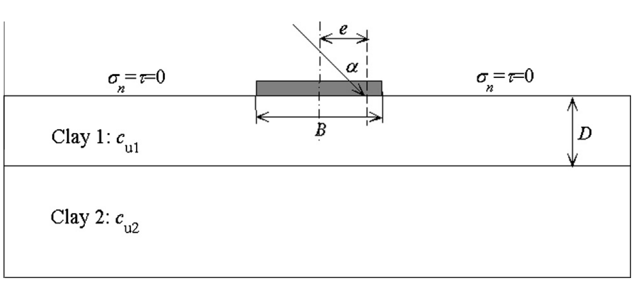

Fig. 1. General layout of the problem.

This covers most problems of practical interest. Note that $c_{u1}/c_{u2} > 1$ corresponds to the common case of a stiff clay layer over a soft clay layer, while $c_{u1}/c_{u2} < 1$ corresponds to the reverse scenario.

The applied load on the strip footing is inclined (characterized by the load inclination α) and eccentric (denoted as the load eccentricity e) with respect to the central line of the footing, as shown in Fig. 1, together with the stress boundary conditions. This loading can also be represented by three statically equivalent forces V ($V = Q{\cdot}\cos\alpha$), H ($H = Q{\cdot}\sin\alpha$), and M ($M = Q{\cdot}e{\cdot}\cos\alpha$), as shown in Fig. 2a. All possible combinations of vertical load V, horizontal load H, and overturning moment M causing a footing to fail delineate the failure envelope, which can be written as a function $f(V, M, H)$. The aim of this paper is to determine the function $f(V, M, H) = 0$, which is represented in terms of the following dimensionless loads

$$v' = \frac{V}{Bc_{u1}}, \quad h' = \frac{H}{Bc_{u1}}, \quad m' = \frac{M}{B^2 c_{u1}} \tag{1a}$$

which indicates the absolute size of the failure envelope, or in terms of normalized loads

$$v = \frac{V}{V_{\text{ult}}}, \quad h = \frac{H}{H_{\text{ult}}}, \quad m = \frac{M}{M_{\text{ult}}} \tag{1b}$$

where V_{ult}, H_{ult}, and M_{ult} represent the ultimate vertical, horizontal, and moment capacity, respectively. These equations indicate the shape and relative size of the failure envelope.

Due to the symmetry of the problem, analyses with $V \geqslant 0$ and $H, M \geqslant 0$ are adequate to define the complete envelope in each of the VH and VM loading planes. Therefore, analyses for inclined and eccentric loadings are only performed for ($V \geqslant 0$, $H \geqslant 0$) and ($V \geqslant 0$, $M \geqslant 0$), respectively. The definition for the positive loading direction is presented in Fig. 2b.

3. Finite element formulation of lower bound limit analysis

The following section is a brief summary of the use of special finite element formulations and second-order cone programming to compute lower bound solutions.

3.1. Statically admissible stress field

The lower bound theorem states that the collapse load calculated from a statically admissible stress field is a lower bound to the actual collapse load. For a stress field to be statically admissible, the following conditions need to be satisfied: (a) equilibrium within each element; (b) continuity of normal and shear stresses along the interface between two adjacent soil elements; (c) equilibrium and/or compatibility at the boundaries; and (d) non-violation of the Tresca criterion in the soil mass. In the event of a general shear failure, the magnitude of the mobilized shear stress τ_t along the footing–soil interface should not exceed the shear strength of the soil mass, namely, $|\tau_t| \leqslant c_{u1}$. The stresses over the loaded segment must satisfy the force and moment equilibrium in the case of eccentric and inclined loading. If no tension can be sustained across the footing–soil interface, separation can occur along the interface under the moment load. In this case, the Tresca yield criterion must be modified to include tension cut-off (i.e., $\sigma_n \leqslant 0$). These considerations were discussed in Ukritchon et al. [28] and Tang et al. [27] and then applied in this study.