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Research Paper

The elastic distortion problem with large rotation in discontinuous deformation analysis

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ABSTRACT

Conventional discontinuous deformation analysis (DDA) results in a change of block volume, which is known as free expansion, during rotation calculations because of the use of a linear displacement function to simulate the behavior of a block with a rigid body and elastic behaviors. This study demonstrates that the linear displacement function also generates unsolved elastic distortion, especially when the block undergoes large rotation in each calculation step. The distortion disturbs the contact judgment in the open–close iteration and update calculations of vertex coordinates, stresses, velocities, etc. at the end of each calculation step. A new procedure follows the flow chart of the original DDA, but it adopts additional codes for the coordinate-transformation calculations in vertex coordinate, stress, and velocity updates. When the vertex coordinates are updated, vertex displacements caused by strains are calculated before involving the block-rotation term in the displacement function to mitigate the elastic distortion. In addition, new codes compile formulas to transform stresses and velocities with block rotation. The new DDA ensures the correctness of rotating elastic calculations to solve practical falling rock problems with a large rotational angle in each calculation step.

1. Introduction

Discontinuous deformation analysis (DDA) is a discrete numerical method [1]. Originally, two-dimensional (2D) DDA used linear displacement function (Eq. (1)) to simulate the behavior of block i. Open–close iteration ensures the correctness of contacts among blocks in each calculation step. The iteration is a unique back-analysis method in DDA to arrange the most suitable contact spring patterns for contact computation in each calculation step. Each contact must satisfy “No penetration” and “No tension.” When blocks penetrate each other, contact spring must be added as no penetration. Conversely, contact spring must be deleted when the contact exceeds the tensile strength of the discontinuity as no tension. The value of time increment will be reduced if the suitable spring patterns are unavailable within six iterations [1]:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -(y-y_0) & (x-x_0) & 0 & \frac{(y-y_0)}{2} \\ 0 & 1 & (x-x_0) & 0 & (y-y_0) & \frac{(x-x_0)}{2} \end{bmatrix} \cdot \begin{Bmatrix} u_0 \\ v_0 \\ r_0 \\ \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = [T_i] \cdot \{D_i\} \qquad (1)$$

where (u, v) are the displacements of an arbitrary point (x, y) in the X and Y directions, (x_0, y_0) are the coordinates of the block centroid, (u_0, v_0) are the rigid-body translations, r_0 is the rigid-body rotation angle (in radians) with a rotation center at (x_0, y_0), $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ are the normal and shear strains, $[T_i]$ is the first-order displacement function, and $\{D_i\}$ is the vector of displacement variables.

In slope disaster mitigation, DDA is a useful tool for investigating the failure mechanism and the impact area because it can trace the rock trajectories and contacts with large displacements [2–4]. After a slope fails, contacts among blocks may cause fast rotation. In addition, DDA applies an implicit solution procedure, called the Newmark-β method for the time integration [1]. Numerically, DDA is unconditionally stable, and it has the advantages of using a larger time interval and fewer calculation steps than an explicit solution procedure [5,6], such as the distinct element method [7]. Therefore, blocks in DDA may rotate largely in each calculation step.

Assume a 4×2-m rectangular block (Fig. 1) with the physical properties as listed in Table 1 is investigated in this study. The negative value of the initial horizontal stress means that it is a compressive stress. The initial time increment is 1.0 s. The force of gravity is neglected to simplify the problem. The elastic deformations $(\varepsilon_x, \varepsilon_y, \gamma_{xy})$ of the block at each calculation step are constrained to (0, 0, 0) by neglecting the impacts of strains to vertex coordinates, stress, and velocity update.

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After solving the problem shown in Fig. 1, a significant volume change of the block due to the large rigid-body rotation angle in each calculation step is called the free-expansion problem in original DDA (Fig. 2). Eq. (2) is the exact solution of the elastic body movement with rotation [8]. The expansion comes from the difference between the nonlinear rotation term, r_0 (Eq. (2)), and the linear displacement function (Eq. (1)):

$$u = u_0 + (x - x_0)(\cos r_0 - 1) - (y - y_0)\sin r_0 + (x - x_0)\varepsilon_x + (y - y_0)\gamma_{xy}/2 \quad (2a)$$

$$v = v_0 + (x - x_0)\sin r_0 + (y - y_0)(\cos r_0 - 1) + (y - y_0)\varepsilon_y + (x - x_0)\gamma_{xy}/2 \quad (2b)$$

The post-adjustment method [8,9], Taylor series method [10], and trigonometric method [11] are techniques available to mitigate the free-expansion problems. However, a new question arises regarding whether the values and the directions of the stresses of the rotating block are constant in conventional DDA. Fig. 3 shows that the stresses of the rotating block are independent of the cumulative rotation angle. The negative value in the vertical axis shown in Fig. 3 indicates the compressive stress. This question is crucial for DDA because a wrong stress direction disturbs the stress distribution and elastic deformation of a block. Therefore, this study focuses on discussing the mechanism of elastic distortions caused by a large rigid-body rotation in DDA, and it proposes a new procedure to mitigate the question.

2. Previous solutions

It is essential to check the correctness of applying the available techniques to mitigate the free expansion, such as the post-adjustment method, Taylor series method [10], and trigonometric method [11] to calculate the stresses as shown in Fig. 1.

2.1. Post-adjustment method

The post-adjustment method uses Eq. (2) to correct the coordinates of the block after open–close iterations [8]. In addition, Koo and Chern [9] used a similar approach to generate the rigid-body DDA.

The unchanged values and the direction of the block stresses at each step calculated by the post-adjustment method (Fig. 4a) are similar to those of original DDA (Fig. 3). In addition, Fig. 4b shows that the width, $L1$, and length, $L2$, shown in Fig. 1, are unchanged, indicating the efficient decrease of free expansion for the elastic-deformation constraint problem.

However, Fig. 5 shows the calculation results of rotating the block without elastic-deformation constraints, where the (ε_x, ε_y, γ_{xy}) after open–close iteration are no longer set as (0, 0, 0) in each calculation step. The following abnormal phenomena are obtained in Fig. 5, and they must be solved as follows:

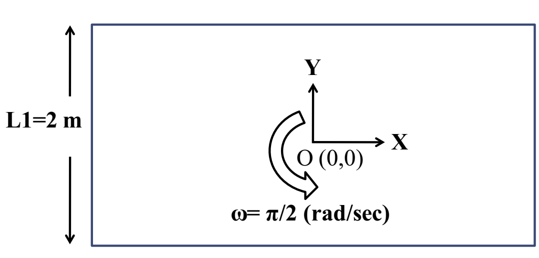

Fig. 1. Geometry of the rectangular block.

Table 1
Physical parameters of the 2D case study.

Item	Value	Item	Value
Block density, ρ (g/cm^3)	2.4	Initial horizontal stress (kPa)	-10^3
Young's modulus, E (MPa)	100	Initial vertical stress (kPa)	0.0
Poisson's ratio, υ	0.0	Initial shear stress (kPa)	0.0
Initial angular velocity, ω (rad/s)	$\pi/2$	Time increment (s)	1.0

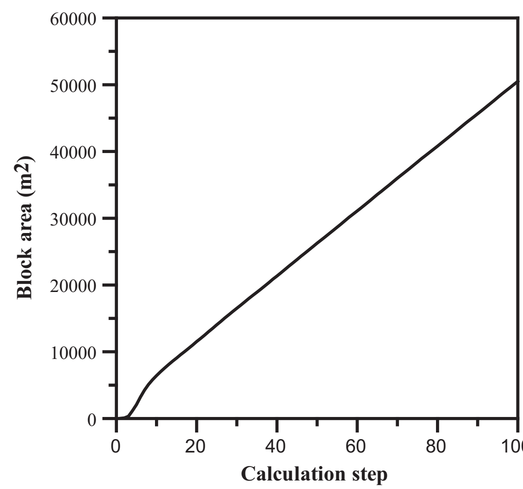

Fig. 2. Free expansion phenomena in original DDA.

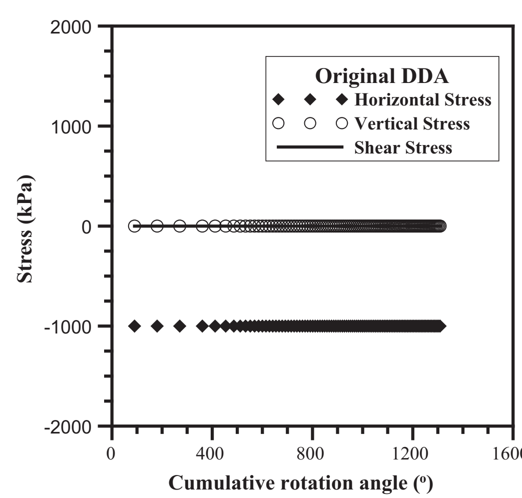

Fig. 3. Stresses calculated by original DDA.

(1) The area difference is defined as the subtraction block area calculation in DDA by simplex integration [1] from ($L1 \times L2$). The area difference is zero when the block is rectangular (Fig. 1). Assume $L2$ is the base. When the rectangle deforms to a parallelogram, the area difference is positive because $L1$ is longer than the height. The variation of the positive area difference at each calculation step (Fig. 5a) implies a block distortion in DDA.

(2) The width of the block, $L1$, shown in Fig. 1, vibrates (Fig. 5b) although the horizontal stress is the unique initial stress for the rotating block calculations, and Poisson's ratio is assumed to be 0.0.