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Research Paper

A numerical model for nonlinear large deformation dynamic analysis of unsaturated porous media including hydraulic hysteresis

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ABSTRACT

A numerical model based on the theory of mixtures is proposed for the nonlinear dynamic analysis of flow and deformation in unsaturated porous media. Starting from the conservation laws, the governing differential equations and the finite element incremental approximations suitable for nonlinear large deformation static and dynamic analyses are derived within the updated Lagrangian framework. The coupling between solid and fluid phases is enforced according to the effective stress principle taking suction dependency of the effective stress parameter into account. The effect of hydraulic hysteresis on the effective stress parameter and soil water characteristic curve is also taken into account. The application of the approach is demonstrated through numerical analyses of several fundamental nonlinear problems and the results are compared to the relevant analytical solutions. The effects of suction, large deformations and hydraulic hysteresis on static and dynamic response of unsaturated soils are particularly emphasized.

1. Introduction

Dynamics of unsaturated, multi-phase porous materials is of interest to many branches of engineering and science including geophysics [1–3], seismology [4–6], acoustics [7,8], biomechanics [9], reservoir engineering [10,11], pavement engineering [12], fracture mechanics [13,14], and materials science [15]. In geotechnical engineering, example applications include: soil–structure interaction analysis [16,17], wave propagation in geological media due to seismic or blast loading [18,19], studies of landslide [20] and liquefaction phenomena [21,22].

Numerous contributions have been made to dynamics of geomaterials in the past few decades. However, they have mostly been limited to the analysis of single-phase fully drained (dry) or two-phase fully saturated porous media; e.g. see [23–26]. This has largely been due to the inherent complexities associated with the behaviour of multi-phase unsaturated porous media, due to simultaneous flow of fluids through the media, interaction among the phases, and strongly nonlinear deformation behaviour of the soil matrix, particularly under large deformations. Zienkiewicz et al. [27] were first to develop a numerical model for the quantitative study of dynamic behaviour of unsaturated soils, which was later extended to finite strains by Meroi et al. [28]. However, the model was based on the extended Biot's theory with the simplifying assumption of constant pore gas pressure, which is shown to alter the dynamic response of the medium, and over-estimate the stability of geo-structures under seismic loading [29]. This deficiency was later rectified by Schrefler and Scotta [30] by introducing pore gas pressure as a primary variable, but in the small deformation framework. Similarly, Muraleetharan and Wei [31] and Ravichandran and Muraleetharan [32] developed the governing equations for dynamic analysis of unsaturated porous media, but used the two stress state theoretical approach rather than a single effective stress approach to cast the constitutive relationships of the soil. A major difficulty with the two stress state approach is that it requires determination of two sets of material parameters, one for each of the stress state variables, which in fact may not be independent, and lead to intractable stress strain relationships. More recently, Uzuoka and Borja [33] presented a neo-Hookean hyper-elastic model for the dynamic behaviour of unsaturated poroelastic solids involving large deformation. The constitutive equations were cast in terms of the second Piola–Kirchhoff stress tensor and the Cauchy–Green deformation tensor, which may be difficult to extend into the highly nonlinear region of soil behaviour. In addition, they used the degree of saturation as the effective stress parameter, which is not supported by the experimental evidence [34,35], and ignored the effect of hydraulic hysteresis, which can markedly alter the response of an

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unsaturated soil during dynamic loading that will invariably involve complex cycles of strain-induced wetting and drying [36–38]. To the authors' knowledge there currently exists no rigorous formulation for nonlinear dynamic analysis of unsaturated porous media suited to large deformation analysis and taking into account all the intricate aspects of behaviour in unsaturated soils.

This paper presents a fully coupled flow-deformation formulation based on the theory of mixtures for rigorous nonlinear dynamic analysis of unsaturated porous media overcoming the deficiencies highlighted above. The finite element incremental solution of the governing equations is derived in large deformation regime with the updated Lagrangian description. The theoretical approach adopted is based on the model proposed by Khalili et al. [39], addressing the suction dependency and volume change of the effective stress parameters. The effective stress is expressed using Bishop's equation, with the effective stress parameter defined using the relationship proposed by Khalili and Khabbaz [40]. The effect of hydraulic hysteresis on the effective stress parameter and soil water characteristic curve is accounted for using the model proposed by Khalili and Zargarbashi [35]. The constitutive equation of the solid phase is described using the Jaumann rate of effective Cauchy stress tensor and the stretching tensor, which are singularly suited to nonlinear analyses of soils. Numerical results and comparisons with known analytical solutions are presented, demonstrating the performance of the proposed approach. The effects of suction, large deformations and hydraulic hysteresis on static and dynamic response of unsaturated soils are particularly emphasized.

2. Governing equations

2.1. Sign convention

The sign convention of continuum mechanics is adopted throughout with tension taken as positive and compression as negative. Pore water and pore air pressures are considered positive in compression, following the soil mechanics convention.

2.2. Volume fraction and density of mixture

Unsaturated soils consist of three phases: solid (S), water (W), and gas (G), which within the context of theory of mixtures are assumed to be continuously distributed throughout representative elementary volume. Each constituent has a volume V_α and a mass $M_\alpha(\alpha = S, W, G)$. The total volume V is obtained from the sum of the partial volumes of the constituents, namely $V = \sum_\alpha V_\alpha$. The apparent mass density of phase α is denoted $\rho^\alpha = M_\alpha/V$, whereas the intrinsic density is written as $\rho_\alpha = M_\alpha/V_\alpha$; hence $\rho^\alpha = n^\alpha \rho_\alpha$, where $n^\alpha = V_\alpha/V$ is the volume fraction of phase α. Finally, the density of the mixture is expressed as $\rho = \sum_\alpha \rho^\alpha = \sum_\alpha \rho_\alpha n^\alpha$.

2.3. Effective and partial Cauchy stress tensors

The effective Cauchy stress tensor for unsaturated soil is expressed as

$$\sigma'_{ij} = \sigma_{net\ ij} - \chi s \delta_{ij} \tag{1}$$

where $\sigma_{net\ ij} = \sigma_{ij} + p_G \delta_{ij}$ is the net Cauchy stress tensor, $s = p_G - p_W$ is the suction, χ is the effective stress parameter, σ_{ij} is the total Cauchy stress tensor, p_G is the pore gas pressure, p_W is the pore water pressure, and δ_{ij} is the Kronecker delta. Following the work of Khalili and Khabbaz [40] and Khalili et al. [41], the effective stress parameter is defined as

$$\chi = \begin{cases} 1 & \text{for } \frac{s}{s_e} \leqslant 1 \\ \left(\frac{s}{s_e}\right)^{-\Omega} & \text{for } \frac{s}{s_e} > 1 \end{cases} \tag{2}$$

where Ω is a material parameter with the best fit value of 0.55, and s_e is the suction value marking the transition between saturated and unsaturated states. For wetting process, s_e is equal to the air expulsion value, s_{ex}, whereas for drying process, s_e is equal to the air entry value, s_{ae}. s_e is a priori a function of the specific volume or volume change of the solid skeleton. This leads to a shift to the right of the effective stress parameter curve and soil water characteristic curve with increasing density [39]. Khalili and Zargarbashi [35] experimentally studied the effect of hydraulic hysteresis on the effective stress parameter, and established the following correlation for suction reversals:

$$\chi = \begin{cases} \left(\frac{s_{rd}}{s_{ae}}\right)^{-\Omega}\left(\frac{s}{s_{rd}}\right)^{\zeta} & \text{for drying path reversal} \quad \left(\frac{s_{ex}}{s_{ae}}\right)^{\frac{\Omega}{\Omega+\zeta}} s_{rd} \leqslant s \leqslant s_{rd} \\ \left(\frac{s_{rw}}{s_{ex}}\right)^{-\Omega}\left(\frac{s}{s_{rw}}\right)^{\zeta} & \text{for wetting path reversal} \quad s_{rw} \leqslant s \leqslant \left(\frac{s_{ae}}{s_{ex}}\right)^{\frac{\Omega}{\Omega+\zeta}} s_{rw} \end{cases} \tag{3}$$

where ζ is the slope of the transition line between the main wetting and main drying paths in a $\ln\chi \sim \ln s$ plane, and s_{rw} and s_{rd} are the points of suction reversal on the main wetting and main drying paths, respectively (Fig. 1(a)). Constitutive relations for soils are highly nonlinear, and are generally expressed in the incremental format. The incremental form of the effective Cauchy stress equation is in turn written as

$$\dot{\sigma}'_{ij} = \dot{\sigma}_{net\ ij} - \psi \dot{s} \delta_{ij} \tag{4}$$

where $\psi = \partial(\chi s)/\partial s$ is the incremental effective stress parameter.

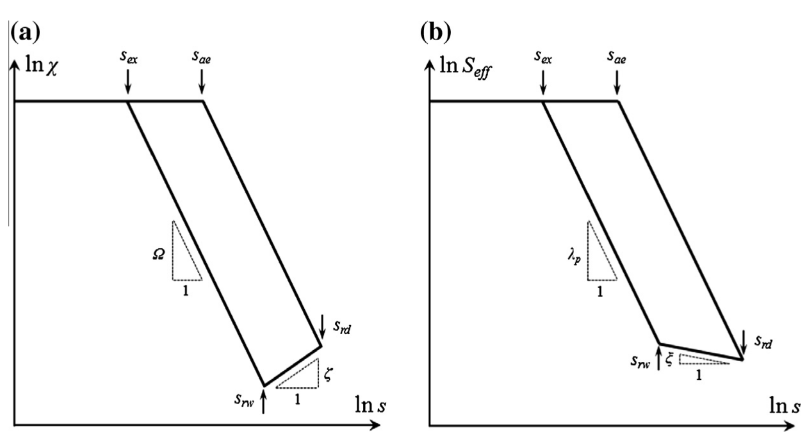

Fig. 1. (a) Evolution of effective stress parameter and (b) soil water characteristic curve including hydraulic hysteresis.