Computers and Geotechnics 69 (2015) 496–505

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Efficient response surface method for practical geotechnical reliability analysis

J. Zhang [a], H.Z. Chen [a], H.W. Huang [a], Z. Luo [b,*]

[a] Key Laboratory of Geotechnical and Underground Engineering of Ministry of Education and Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China
[b] Department of Civil Engineering, University of Akron, Akron, OH 44325, USA

ARTICLE INFO

Article history:
Received 2 February 2015
Received in revised form 10 June 2015
Accepted 11 June 2015

Keywords:
Reliability
Response surface method
Shallow foundation
Deep mixed columns

ABSTRACT

Although numerical models have been widely used in the geotechnical profession, their applications for reliability analysis are still rather limited mainly because most geotechnical numerical programs lack the probabilistic function. In this study, an efficient response surface method is suggested for geotechnical reliability analysis using existing numerical programs. The developed approach can effectively avoid the occurrence of negative values for positive random parameters, and thus it solves the key limitation of the classical response surface method in geotechnical applications. To facilitate its application, a procedure is designed for automating reliability analysis with the commercially available program, FLAC3D. The developed method and procedure are demonstrated in detail using a slope example. The versatility of the suggested procedure is illustrated through the serviceability reliability analysis of a soft ground improved with deep mixing columns. For the reinforced ground studied in this paper, the reliability is mainly governed by the uncertainties in Young's moduli of the soil and reinforced columns. The suggested method can be conveniently used for reliability-based design of deep mixing columns.

1. Introduction

To assess the effect of uncertainties on geotechnical predictions, reliability methods are increasingly being adopted in geotechnical engineering (e.g., Refs. [1,2]). Several geotechnical design codes have been developed based on the reliability theory (e.g., Refs. [3–5]). Probably because most geotechnical numerical programs do not have the probabilistic analysis function, the applications of reliability methods are still limited to problems with relatively simple limit state functions. Compared with the wide application of numerical models in geotechnical engineering, the lack of ability for reliability analysis based on numerical models greatly limits the value of reliability analysis in geotechnical engineering. How to implement reliability analysis utilising sophisticated geotechnical numerical models has been one of the key challenges for the application of reliability methods in geotechnical engineering.

Efforts have been dedicated to the development of specific numerical programs capable of geotechnical reliability analysis (e.g., Refs. [6,7]). Such programs can profoundly facilitate the applications of reliability methods for complex geotechnical problems. However, developing such numerical programs requires expertise in both geotechnical numerical analysis and geotechnical reliability. In this regard, response surface methods (RSM) as alternatives are adopted for complex geotechnical reliability problems, in which the reliability analysis is realised through the computationally efficient models that approximate the deterministic numerical solutions (e.g., Refs. [8–15]). Among the RSM available, the iterative method suggested by Bucher and Bourgund [16] based on the first-order reliability method (FORM) is shown to be quite efficient and thus commonly used in various fields (e.g., Refs. [17–22]). The method by Bucher and Bourgund [16] is termed the classical RSM in this paper. As will be shown subsequently in this study, the classical RSM may be subjected to serious convergence difficulties and may not be suitable for geotechnical applications.

The objective of this paper is to suggest an efficient and robust RSM for geotechnical reliability analysis that can overcome the convergence difficulty of classical RSM. This method can automate reliability analysis using commercial geotechnical programs such as FLAC3D [23]. As such, the suggested method can profoundly extend the capability of the profession for reliability analysis of complex geotechnical problems. The structure of this paper is as follows: First, FORM is reviewed and the limitation of the classical RSM is discussed. Then, a modified RSM is introduced to deal with

* Corresponding author.
E-mail addresses: cezhangjie@gmail.com (J. Zhang), emailchz@gmail.com (H.Z. Chen), huanghw@tongji.edu.cn (H.W. Huang), zluo@uakron.edu (Z. Luo).

http://dx.doi.org/10.1016/j.compgeo.2015.06.010
0266-352X/

the limitation of the classical RSM. Thereafter, a procedure for automating geotechnical reliability analysis using FLAC3D based on the suggested RSM is described. Finally, the applicability and effectiveness of the suggested method are illustrated through the serviceability reliability analysis of a soft ground reinforced with deep mixed (DM) columns.

2. First-order reliability method (FORM)

Let **x** denote the uncertain variables in the performance function $g(\mathbf{x})$ with $g(\mathbf{x}) < 0$ indicating failure. Various methods such as FORM (e.g., Ref. [24]) and Monte Carlo simulation (e.g., Ref. [25]) are available for the estimation of failure probability, defined as the probability of $g(\mathbf{x}) < 0$. Although Monte Carlo simulation is one of the most flexible and versatile approaches, its application is often challenged by the considerable computational work involved. For comparison, as FORM is reasonably accurate and also computationally efficient, it is applicable to a large number of practical problems and widely used in practice [26]. In this paper, the focus is on geotechnical reliability analysis using solutions of numerical models, and FORM is employed to achieve computational efficiency. With FORM, the failure probability (p_f) is calculated as follows (e.g., Ref. [24]):

$$p_f = 1 - \Phi(\beta) \tag{1}$$

$$\beta = \min_{g(\mathbf{x})=0} \sqrt{\mathbf{y}\mathbf{R}^{-1}\mathbf{y}^T} \tag{2}$$

where Φ = cumulative distribution function (CDF) of the standard normal variable, β = reliability index, **y** = reduced variables of **x**, and **R** = correlation matrix of the random variables.

FORM can be efficiently executed in a spreadsheet when the limit state function has an explicit form [24]. When the explicit solution of a problem is not available and it has to be evaluated with a stand-alone numerical program, the implementation of FORM is challenging due to the coupling between the numerical program and FORM. To deal with such a challenge, Bucher and Bourgund [16] suggested a FORM-based RSM for conducting reliability analysis based on computationally expensive deterministic models, which is now widely employed in various fields. As mentioned previously, the method suggested by Bucher and Bourgund [16] is termed the classical RMS in this study.

3. Classical RSM and its limitation

The classical RSM approximates the performance function $g(\mathbf{x})$ by a second-order polynomial function:

$$g(\mathbf{x}) \approx b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{k+i} x_i^2 \tag{3}$$

where x_i = the ith element of **x**, k = dimension of **x**, and b_i ($i = 0, 1, \ldots, 2k$) = unknown *deterministic* coefficients. To determine the $(2k + 1)$ unknown coefficients, the performance function can be first evaluated around a centre point $\mathbf{x}_c = \{x_{c1}, x_{c2}, \ldots, x_{ck}\}$ and other $2k$ points around $\mathbf{x}_c$: $\{x_{c1} \pm m\sigma_{x1}, x_{c2}, \ldots, x_{ck}\}$, $\{x_{c1}, x_{c2} \pm m\sigma_{x2}, \ldots, x_{ck}\}, \ldots,$ and $\{x_{c1}, x_{c2}, \ldots, x_{ck} \pm m\sigma_{xk}\}$, where m is a parameter determining the relative distance of the calibration points and σ_{xi} = standard deviation of x_i. Equating the values of the performance function with those calculated using Eq. (3) at the prescribed $(2k + 1)$ calibration points, the unknown coefficients can then be solved.

After the second-order polynomial function is established, the reliability index can be calculated with Eq. (3) instead of the numerical model. Let $\mathbf{x}_D$ denote the design point found based on Eq. (3) using FORM. When the performance function is not well approximated by Eq. (3), $\mathbf{x}_D$ computed by Eq. (3) may not be close to the design point of the actual performance function. In such a case, the response surface needs to be updated with a *new* set of calibration points around a new centre point determined using the following equation:

$$\mathbf{x}_c = \boldsymbol{\mu}_\mathbf{x} - g(\boldsymbol{\mu}_\mathbf{x}) \frac{\boldsymbol{\mu}_\mathbf{x} - \mathbf{x}_D}{g(\boldsymbol{\mu}_\mathbf{x}) - g(\mathbf{x}_D)} \tag{4}$$

where $\boldsymbol{\mu}_\mathbf{x}$ = mean of **x**. Through the updating given by Eq. (4), the sampling points are expected to move closer to the limit state function $g(\mathbf{x}) = 0$, indicating a more accurate search for the design point. With the new response surface (Eq. (3) with *updated* coefficients), the reliability index can be recalculated. This updating procedure is iterated until the resulting FORM reliability index does not change within a tolerable error ε_β such as $\varepsilon_\beta = 0.01$.

The above RMS has extensive applications in many fields (e.g., Refs. [17–22]). When determining the calibration points in the classical RSM, however, negative values may occur for positive random variables [21]. Such a phenomenon often occurs when the coefficient of variation (COV) of the random variables is large or when the failure probability is small. In geotechnical engineering, many random variables, such as the cohesion and the friction angle, are non-negative engineering properties. In addition, the uncertainties involved in geotechnical engineering are often higher than those in other fields [27]. Hence, the occurrence of negative values for positive random variables when generating the calibrating points using the classical RSM is quite common in geotechnical engineering, which has previously been noticed by Mollon et al. [21]. Such physically impermissible values result in the iteration termination of numerical programs before the reliability index converges. Therefore, the classical RSM may not be suitable for geotechnical reliability analysis. Although the step size during the iteration process can be adjusted manually to avoid negative values for positive random variables [21], such a procedure has to be realised in a trial-and-error manner and may not always be effective.

4. Modified response surface method

Let **y** denote the reduced variables of **x**, and let $\mathbf{x} = T(\mathbf{y})$ denote the transformation relationship between reduced variables and original variables. Substituting this relationship into $g(\mathbf{x})$, the performance function can be expressed in terms of **y** as follows:

$$g(\mathbf{x}) = g[T(\mathbf{y})] = G(\mathbf{y}) \tag{5}$$

where $G(\mathbf{y})$ is the performance function in the reduced space. A set of transformation equations for different types of random variables is documented in the study by Low and Tang [24]. For instance, if x_i is a lognormal random variable with a mean of μ_{xi} and a COV of δ_{xi}, it can be related to its reduced variable y_i as follows:

$$x_i = T(y_i) = \exp(\lambda_i + \xi_i y_i) \tag{6}$$

where λ_i and ξ_i are the mean and standard deviation of $\ln x_i$, respectively, which can be calculated with the following equations:

$$\lambda_i = \ln \mu_{xi} - 0.5\xi_i^2 \tag{7}$$

$$\xi_i = \sqrt{\ln(1 + \delta_{xi}^2)} \tag{8}$$

As shown in Eq. (6), x_i is always non-negative as assumed in the lognormal distribution regardless of the value of y_i. This procedure is also suitable for other types of non-negative random variables. Thus, if the calibration points are determined in the reduced space instead of the original space, the physically impermissible sampling points can be effectively avoided. With this realisation, we suggest that one can approximate the performance function in the reduced space as follows: