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Research Paper

Efficient reliability updating of slope stability by reweighting failure samples generated by Monte Carlo simulation

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ABSTRACT

Direct Monte Carlo simulation (MCS) could be prohibitively expensive for slope reliability updating problem with relatively low-probability level. This paper aims to propose an efficient slope reliability updating method based on the direct MCS. First, the direct MCS is briefly introduced. Thereafter, a weighted approach based on the direct MCS is proposed for slope reliability updating. Furthermore, the implementation procedure and the flow chart of the proposed method are presented. Two illustrative examples are investigated to demonstrate the capacity and validity of the proposed method. The results indicate that the proposed method can properly utilize the information contained in failure samples generated by performing direct MCS based on the baseline distribution, which is much more efficient than repeatedly performing direct MCS as the probability distributions of soil properties are updated. The proposed method can solve the slope reliability updating problems involving multiple uncertain parameters and implicit performance functions. The accuracy of the proposed method depends on the failure samples generated from the baseline distribution. When the failure samples cannot completely cover the failure region underlying the reliability updating problem, the proposed method should be used with caution. The coefficient of variation (COV) of probability of failure from the proposed approach is derived and used as a measure of the accuracy in updated probability of failure. In geotechnical engineering practices, the updated COV of an uncertain parameter is often smaller than its baseline COV. In this case, the proposed method can reasonably update the probability of failure.

1. Introduction

Slope reliability has been investigated extensively in the literature due to unavoidable uncertainties in geotechnical engineering (e.g., [10,15,14,22,30,45,19,20,17,23,21,36–38,24,25,27]). For slope reliability analysis, the uncertainties in geotechnical parameters involved in slope reliability analysis are often represented by a vector of random variables, $\mathbf{x}$. The statistics of the random variables $\mathbf{x}$ (such as the mean, standard deviation, and correlation coefficient) are usually determined based on the site information from field or laboratory tests as well as engineering judgment available prior to the project. Based on the statistics of $\mathbf{x}$, the joint probability density function (PDF), $f(\mathbf{x})$, of the random variables $\mathbf{x}$ can be constructed, which is referred to as the baseline distribution in this paper. Then, the probability of slope failure can be given by $P_f = \int_{\Omega_{\text{fail}}} f(\mathbf{x})d\mathbf{x}$, in which Ω_{fail} is the failure domain [6,26]. As the project proceeds, slope engineers may obtain additional information about the uncertain parameters. Using the additional information, the joint PDF, $f(\mathbf{x})$, can be updated [32,8,33,27] to obtain a new joint PDF of random variables $\mathbf{x}$, $f^{new}(\mathbf{x})$. Then, there exists a need to re-evaluate/update the slope reliability based on $f^{new}(\mathbf{x})$. Such a problem is a typical slope reliability updating problem resulting from the uncertainty updating of geotechnical parameters involved in slope reliability analysis. Recently, the slope reliability updating problem has been received more and more attentions [33,28].

In the literature, many reliability methods have been developed for the reliability analysis of geotechnical engineering problems, such as direct Monte Carlo simulation (MCS) (e.g., [1]), first order second moment method (FOSM) (e.g., [6]), first order reliability method (FORM) (e.g., [16,29]), second order reliability method (SORM) [12], response surface method (RSM) [22], importance sampling method (IS) [3,9], and Subset Simulation method (SS) [2,5]. Among these reliability methods, direct MCS has the

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advantage of robustness and conceptual simplicity, which has been widely used in reliability analyses and designs of geotechnical structures, such as slopes [4,14,36,41,21], retaining walls [37], and foundations [39,40,43,46].

As pointed out by several researchers [6,11,34,5], direct MCS suffers from a major disadvantage of a lack of computational efficiency. Its computational costs may be too high to be acceptable in design practice, particularly at small probability levels. This problem becomes more profound when reliability updating problems are in question because direct MCS needs to be repeatedly performed as the probability distribution of soil properties are updated. To address the above problem, it is necessary to develop an efficient reliability updating method for slope reliability updating.

The objective of this study is to propose an efficient slope reliability updating method based on the direct MCS. To achieve this goal, this paper is organized as follows. In Section 2, the direct MCS is briefly introduced first. Then, a weighted approach based on the direct MCS is developed for solving slope reliability updating problems. Thereafter, the coefficient of variation (COV) of probability of failure is derived to measure the accuracy of the proposed method. The implementation procedure and the flow chart of the proposed method are presented in Section 3. In Section 4, the proposed method is illustrated using two slope examples.

2. A new efficient slope reliability updating method based on direct MCS

A new efficient reliability updating method based on direct MCS is developed in this study. The proposed slope reliability updating method generally contains two major steps as below. The first step is to perform direct MCS using the baseline joint PDF of random variables involved in slope reliability analysis. The corresponding failure samples generated from the direct MCS are recorded, which will be used to conduct slope reliability updating in the second phase. The second step is to update the slope reliability on the basis of updated joint PDF of random variables and failure samples generated by direct MCS in the first step. The two steps are further discussed in the following two subsections, respectively.

2.1. Direct Monte Carlo simulation

Let $\mathbf{x}$ be the vector of random variables involved in slope reliability analysis. Its joint PDF is denoted by $f(\mathbf{x})$. The performance function for slope reliability analysis is usually defined as [10]

$$g(\mathbf{x}) = FS(\mathbf{x}) - 1.0 \tag{1}$$

where FS is factor of safety for a slope. If $g(\mathbf{x}) \leqslant 0$, the slope failure occurs; otherwise, the slope remains stable. The probability of slope failure, P_f, is often expressed as

$$P_f = \int_{g(\mathbf{x})\leqslant 0} f(\mathbf{x})d\mathbf{x} = \int I[g(\mathbf{x})]f(\mathbf{x})d\mathbf{x} = E_{f(\mathbf{x})}\{I[g(\mathbf{x})]\} \tag{2}$$

where $I[g(\mathbf{x})]$ is the indicator function with $I[g(\mathbf{x})] = 1$ for $g(\mathbf{x}) \leqslant 0$ and $I[g(\mathbf{x})] = 0$ for $g(\mathbf{x}) > 0$. Direct MCS provides an effective way to solve the above integral equation [21]. Using direct MCS, the probability of failure is estimated as

$$\widehat{P}_f = \frac{1}{N}\sum_{i=1}^{N} I[g(\mathbf{x}_i)] \tag{3}$$

in which $\widehat{P}_f$ is an unbiased estimator of P_f; $\mathbf{x}_i$ ($i = 1, 2, \ldots, N$) are random samples of $\mathbf{x}$ generated from the joint PDF $f(\mathbf{x})$; N is the sample size.

During the process of the direct MCS, the samples with $FS \leqslant 1.0$ are defined as failure samples, which have a significant effect on $\widehat{P}_f$. Let n_s be the number of failure samples, $\widehat{P}_f$ can be further expressed as

$$\widehat{P}_f = \frac{1}{N}\sum_{j=1}^{n_s} I[g(\mathbf{x}_j)] = \frac{n_s}{N} \tag{4}$$

where $\mathbf{x}_j$ ($j = 1, 2, \ldots, n_s$) are the failure samples. It should be pointed out that the uncertainty of $\widehat{P}_f$ obtained from the direct MCS is commonly measured by its variance or COV. The variance of $\widehat{P}_f$ is given by [1]

$$Var[\widehat{P}_f] \approx \frac{1}{N-1}(\widehat{P}_f - \widehat{P}_f^2) \tag{5}$$

Thus, the COV of $\widehat{P}_f$ can be expressed as

$$COV[\widehat{P}_f] = \frac{\sqrt{Var[\widehat{P}_f]}}{E[\widehat{P}_f]} \approx \sqrt{\frac{1-\widehat{P}_f}{(N-1)\widehat{P}_f}} \tag{6}$$

$COV[\widehat{P}_f]$ can measure the accuracy of MCS effectively. As can be seen from Eq. (6), $COV[\widehat{P}_f]$ decreases with the increase of the sample size. For a prescribed target $COV[\widehat{P}_f]$, direct MCS requires, at least, a total of $\frac{1-\widehat{P}_f}{\widehat{P}_f(COV[\widehat{P}_f])^2}$ to ensure the desired accuracy (e.g., [1]). For example, as the target $COV[\widehat{P}_f] < 0.1$, it requires, at least, 10^5 random samples in direct MCS for evaluating a probability of failure up to 10^{-3}.

2.2. Weighted approach based on direct MCS

Based on the baseline distribution $f(\mathbf{x})$, the probability of slope failure can be evaluated first. The corresponding failure samples can also be obtained. For slope engineering, more information related to slope project will be collected along with the slope project underway. Such information can be used to update the uncertainties in random variables. Thus, a new joint PDF of considered random variables, $f^{new}(\mathbf{x})$, will be obtained, which will be further used to update the probability of slope failure. The updated probability of slope failure, P_f^{new}, can be derived as

$$\begin{aligned} P_f^{new} &= \int_{g(\mathbf{x})\leqslant 0} f^{new}(\mathbf{x})d\mathbf{x} = \int_{g(\mathbf{x})\leqslant 0} \frac{f^{new}(\mathbf{x})}{f(\mathbf{x})}f(\mathbf{x})d\mathbf{x} \\ &= \int I[g(\mathbf{x})]\frac{f^{new}(\mathbf{x})}{f(\mathbf{x})}f(\mathbf{x})d\mathbf{x} = E_{f(\mathbf{x})}\{I[g(\mathbf{x})]\omega\} \end{aligned} \tag{7}$$

where the weighting index, ω, is defined as

$$\omega = \frac{f^{new}(\mathbf{x})}{f(\mathbf{x})} \tag{8}$$

It is evident that the weighting index only depends on the baseline and updated distributions of $\mathbf{x}$. According to the weighting ideas [7,31], the samples generated based on $f(\mathbf{x})$ in Section 2.1 are reused to evaluate the updated probability of slope failure, which is derived as

$$\widehat{P}_f^{new} = \frac{1}{N}\sum_{i=1}^{N}\left(I[g(\mathbf{x}_i)]\frac{f^{new}(\mathbf{x}_i)}{f(\mathbf{x}_i)}\right) = \frac{1}{N}\sum_{i=1}^{N}(I[g(\mathbf{x}_i)]\omega_i) \tag{9}$$

Note that $\mathbf{x}_i$ ($i = 1, 2, \ldots, N$) are the samples generated using the baseline distribution $f(\mathbf{x})$ in Section 2.1. The value of indicator function $I[g(\mathbf{x}_i)]$ has been obtained in Section 2.1. In this way, the results obtained from the direct MCS in the previous subsection can be sufficiently used for calculating the updated probability of slope failure $\widehat{P}_f^{new}$. In addition to the results obtained from the direct MCS, the weighting index need to be determined, which