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Research Paper

A fractal model for characterizing fluid flow in fractured rock masses based on randomly distributed rock fracture networks

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ABSTRACT

A fractal model that represents the geometric characteristics of rock fracture networks is proposed to link the fractal characteristics with the equivalent permeability of the fracture networks. The fracture networks are generated using the Monte Carlo method and have a power law size distribution. The fractal dimension D_T is utilized to represent the tortuosity of the fluid flow, and another fractal dimension D_f is utilized to represent the geometric distribution of fractures in the networks. The results indicate that the equivalent permeability of a fracture network can be significantly influenced by the tortuosity of the fluid flow, the aperture of the fractures and a random number used to generate the fractal length distribution of the fractures in the network. The correlation of fracture number and fracture length agrees well with the results of previous studies, and the calculated fractal dimensions D_f are consistent with their theoretical values, which confirms the reliability of the proposed fractal length distribution and the stochastically generated fracture network models. The optimal hydraulic path can be identified in the longer fractures along the fluid flow direction. Using the proposed fractal model, a mathematical expression between the equivalent permeability K and the fractal dimension D_f is proposed for models with large values of D_f. The differences in the calculated flow volumes between the models that consider and those that do not consider the influence of fluid flow tortuosity are as high as 17.64–19.51%, which emphasizes that the effects of tortuosity should not be neglected and should be included in the fractal model to accurately estimate the hydraulic behavior of fracture networks.

1. Introduction

Permeability is a crucial hydro-mechanical property of rock masses and is important in many areas of geosciences and geoengineering, including dam foundations and petroleum reservoirs. The permeability of a rock mass is mainly governed by rock fractures that separate intact rock blocks with negligible matrix permeability (e.g., granite and basalt) [1,2]. A tremendous amount of effort has been exerted to understand the behavior of fluid flow in rock masses in recent decades [3–7]. However, accurately estimating the permeability of rock masses is still challenging because of the complexities of fracture distributions at the macro-structural (i.e., geometry of the fracture network) and micro-structural (i.e., geometry of the void spaces within single fractures) levels [8–11]. One key difficulty is that rock fractures typically have rough surfaces between which fluid flows non-uniformly. A particle will travel a longer distance along a tortuous path through a rough-walled fracture than through a parallel-walled fracture. Another difficulty is mathematically describing the geometric distributions of rock fractures in fracture networks, which usually contain several sets of fractures with different orientations, lengths and apertures. Fortunately, the distribution of fractures in fracture networks have been found to exhibit fractal characteristics [12–16], which provides a possible approach for describing the geometric characteristics of fracture networks while considering both the macro-scale and micro-scale properties of the fractures.

A few predictive fractal models have been developed to calculate the permeability of stochastic rock fracture networks. The purpose of their models and the outcome of their studies are summarized in Table 1.

Based on the fractal models proposed for the porous media [10] and regular tree networks [21,27] to calculate the permeability of

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rock masses, the present study focused on extending this fractal model to the fractured media consisted of randomly generated stochastic discrete fracture networks (DFNs) using the Monte Carlo method. A probability density function was derived to depict the trace length l of each rock fracture between a minimum and maximum trace length, and the apertures of the fractures were correlated with their trace lengths. Flow simulations of models with various fractal dimensions were conducted, and the relations between the fractal dimensions and the equivalent permeability were estimated.

2. Fractal characteristics of rock fractures

Mandelbrot [28] verified that the cumulative size distribution of islands on the surface of the earth followed the power law

$$N'(A' > a') \sim a'^{(-D/2)} \tag{1}$$

where N' is the total number of islands with an area A' greater than a constant a' and D is the fractal dimension that represents the size distribution of the islands.

Based on this theory, Majumdar and Bhushan [29] developed an equivalent equation to describe the distribution of islands by regarding a'_{max} as the largest island:

$$N'(A' \geq a') = \left(\frac{a'_{max}}{a'}\right)^{D/2} \tag{2}$$

Eq. (2) shows that there is only one largest island on the earth, which is true in the physical world. Xu et al. [21,27] used Eq. (2) to describe the geometric distribution of pores in porous media that are embedded with randomly distributed 2-D fractal-like tree networks, where $a'_{max} = g\lambda^2_{max}$, $a' = g\lambda^2$, λ is the diameter of a pore, and g is a geometric factor. The distribution of fractures in 2-D rock masses is considered to be analogous to that of islands on the surface of the earth and that of pores in porous media, which yields

$$N(L \geq l) = \left(\frac{l_{max}}{l}\right)^{D_f/2} \tag{3}$$

where N is the total number of fractures with a length L greater than a constant fracture length l; D_f is in the range of [1,2] for 2-D frac-

Table 1
Review of fractal DFN models used to calculate the permeability of rock masses.

Authors	Year	Purpose of the model	Outcome of the study
De Dreuzy et al. [17–19]	2001a; 2001b; 2002	Investigation of the hydraulic properties of 2-D fracture networks with random fracture geometries that follow a power law length distribution	They analyzed the influence of the power law exponent a in their models and found that if a was greater than 3, the classical percolation model based on a population of small fractures was applicable, and the fluid flow appeared to be relatively homogeneous in the flow direction In contrast, if a was less than 2, the applicable model was made up of the largest fractures of the network, and the main flow paths were composed of a few large fractures Between the two limits ($2 < a < 3$), relatively uniform fluid flow occurred in all of the fractures
Yu and Cheng [10]	2002	Development of a fractal model to calculate the equivalent permeability of bi-dispersed porous media	They extensively evaluated the influences of the fractal dimension D_f (which represents the fracture distribution) and D_T (which represents tortuosity) on the equivalent permeability They found that the equivalent permeability was a function of the tortuous fractal dimension, pore area fractal dimension, sizes of particles and clusters, microporosity inside clusters, and the effective porosity of a medium
Yu et al. [20]	2005	Establishment of a 2-D fractal model to calculate the permeability of a porous media model generated by the Monte Carlo method	Their model can predict the transport properties (i.e., permeability, thermal conductivity, dispersion coefficient and electrical conductivity) of saturated or unsaturated fractal porous media
Xu et al. [21]	2006	Development of a fractal model for fluid flow in porous media that are embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [22] and Bejan and Zane [23]	They found that the permeability of the model that incorporated the flow tortuosity was approximately 20% lower than the model that did not consider the tortuosity
Zou et al. [24]	2007	Establishment of a fractal model to analyze the 3-D surfaces of rock fractures	Their results indicated that larger values of the fractal dimension D of the profile of a rough surface or smaller values of the scaling constant G signify a smoother surface topography
Jafari and Babadagli [25]	2012	Estimation of the equivalent permeability of fracture networks using numerical simulations	They derived a nonlinear multivariable regression to address the equivalent permeability by calculating three parameters: $X1$ (the connectivity index), $X2$ (the box-counting fractal dimension of the fracture lines) and $X3$ (the hydraulic conductivity)
Zheng and Yu [26]	2012	Development of a fractal model for fluid flow in porous media that are embedded with randomly distributed fractal-like tree networks using the constructal theory proposed by Bejan and Lorente [22] and Bejan and Zane [23]	They derived an analytical expression for the gas permeability in dual-porosity media based on the pore size of the matrix and the diameter of the mother channel of embedded fractal-like tree networks They found that for a certain fracture network, the dimensionless permeability K^+, which is defined as the ratio of the porous matrix permeability K_m to the fracture network permeability K_f, increases with increasing matrix porosity ε