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Research Paper

A micromechanical model of inherently anisotropic rocks

W.Q. Shen, J.F. Shao *

LML, UMR CNRS 8107, University of Lille, 59655 Villeneuve d'Ascq, France

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ABSTRACT

A micromechanical elastoplastic model is proposed in this study for anisotropic sedimentary rocks. With a two-step homogenization procedure, a macroscopic criterion is established for geomaterials made up of a porous matrix reinforced by rigid inclusions. The effects of porosity, inclusions and the inherent anisotropy are explicitly taken into account. Based on [18], a scalar anisotropy parameter is introduced to describe the anisotropic characteristics of the material. With a non-associated plastic flow rule and a strain hardening law, the proposed model is applied to describe the macroscopic mechanical behavior of Tournemire shale with different initial confining pressures and different loading orientations. Comparing numerical results with experimental data, the efficiency of the proposed micromechanical model is assessed and verified.

1. Introduction

The inherent anisotropy of rocks is an important property which should be taken into account in stability and safety analysis of structures for various engineering applications. This property is related to specific features of material fabric, such as bedding, layering, crack pattern, etc. A number of experimental studies have been performed to characterize the directional dependence of rock strength ([5,13,2,10,7,15], etc.). Generally, the maximum axial compressive strength is obtained when the loading orientation is either parallel or perpendicular to the bedding planes while the minimum one for a loading orientation between 30° and 60° with respect to the bedding planes. At the same time, various anisotropic models were also proposed for the description of failure criteria and plastic deformation with different methods (for example [31,11,17,8,9,16], direct extension of isotropic criteria by introducing variation of some parameters with loading orientation, using the concept of discontinuous weakness planes, etc.). Based on the observations of microstructure of anisotropic materials [20] proposed an another method. The formulation incorporates a scalar anisotropy parameter which is expressed in terms of mixed invariants of the stress and structure-orientation tensors. This approach was then used in [18] for the complete formulation of an elastoplastic model for anisotropic sedimentary rocks. The model retains the mathematical rigor and remains pragmatic for engineering applications. However, this macroscopic model was developed for homogeneous rocks without considering effects of pores and mineralogical heterogeneities. On the other hand, in the context of geological disposal of radioactive wastes, various clayey rocks have been investigated as a possible geological barrier. It was found that the macroscopic mechanical behaviors of these rocks strongly depend on the mineralogical compositions (mineral grains, pores, etc). Therefore, during the recent years, various micromechanical models have been developed based on homogenization techniques. For instance, the authors of [1,25,27,4,26] have proposed some micro–macro models which explicitly relates the macroscopic mechanical properties to the mineral composition and the porosity for Callovo-Oxfordian argillite or cement based materials. However, all these models are limited to isotropic materials. The objective of this paper is then to extend the micromechanical model proposed by Shen et al. [25] to anisotropic rocks. The extended model will be used to describe the elastoplastic behavior of anisotropic sedimentary rocks (such as Tournemire shale) considering the effects of mineral grains and the influence of pores and also the inherent anisotropy.

The paper is organized as follows. In Section 2, a micromechanical constitutive model will be firstly established for anisotropic materials. With a two-step homogenization procedure, a macroscopic criterion is proposed which takes explicitly the effects of porosity and inclusions, also the influence of inherent anisotropy of the studied material. Then the model is completed by a macroscopic plastic potential and a plastic hardening law. The non-associated model is applied in Section 3 to describe the macroscopic mechanical behavior of Tournemire shale. The numerical results are compared with the available experimental data.

* Corresponding author.
E-mail address: jian-fu.Shao@polytech-lille.fr (J.F. Shao).

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2. Formulation of a micro–macro constitutive model for anisotropic materials

In this section, a micro–macro constitutive model will be established for the anisotropic sedimentary rocks taken from the Tournemire site in the Massif Central, France. According to [15,30], the Tournemire shale is approximately composed of 40–50% of clay minerals, 10–30% of calcite and 10–20% of quartz. The total porosity is between 6.50% and 7.10% which is mainly in the clay matrix. Therefore, as a first approximation, the morphology of the Tournemire shale at the meso-scale can be seen as a porous matrix – particle system. For the sake of simplicity and due to the fact that the elastic properties of calcite and quartz are close each other, like [25] for Callovo-Oxfordian argillite, the quartz and calcite grains will be seen as one family of inclusions, which are spherical and elastic, randomly imbedded in the porous clay matrix. At the microscopic scale, the solid phase of the porous clay matrix is modeled as an anisotropic elastoplastic medium, due to the presence of a set of bedding planes. The representative volume element (RVE) is illustrated in Fig. 1. In the present work, we consider that the conventional sample size, for instance 37 mm × 75 mm, is large enough to define the RVE of the Tournemire shale compared with the size of heterogeneities (mineral grains) which is less than 1 mm. However, due to the strain localization which generally occurs in rocks, the tested sample cannot be considered as the RVE of material after the strain localization. In this present paper, this issue is not discussed and will be considered in future works.

In this RVE Ω, the volume fraction of pores, rigid particles and the solid matrix are respectively denoted as: Ω_p, Ω_i and Ω_m. The porosity f of the porous medium and the absolute volume fraction ρ of particles are:

$$f = \frac{\Omega_p}{\Omega_p + \Omega_m}, \quad \rho = \frac{\Omega_i}{\Omega_i + \Omega_m + \Omega_p} \tag{1}$$

2.1. Description of the inherent anisotropy

Inspired by the works of [20,19], the failure criterion of the anisotropic materials can be expressed in a general form:

$$\Phi(\boldsymbol{\Sigma}, f, \rho, \boldsymbol{a}) = 0 \tag{2}$$

where $\boldsymbol{\Sigma}$ denotes the macroscopic stress, f is the porosity in the porous matrix and ρ the volume fraction of particles, $\boldsymbol{a}$ is a microstructure tensor with its three principal values (a_1, a_2, a_3) in the principal triad $(\underline{S}_1, \underline{S}_2, \underline{S}_3)$. Consider the principal triad of the microstructure tensor $\boldsymbol{a}$ and specify the traction moduli on the planes normal to principal axes. The magnitudes of the traction moduli are:

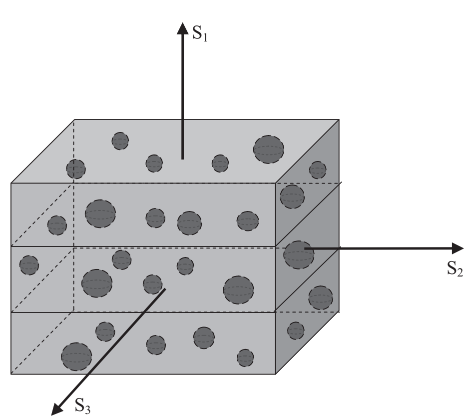

Fig. 1. Representative volume element of Tournemire shale.

$$L_1 = \left(\Sigma_{11}^2 + \Sigma_{12}^2 + \Sigma_{13}^2\right)^{1/2}; \quad L_2 = \left(\Sigma_{12}^2 + \Sigma_{22}^2 + \Sigma_{23}^2\right)^{1/2};$$
$$L_3 = \left(\Sigma_{13}^2 + \Sigma_{23}^2 + \Sigma_{33}^2\right)^{1/2} \tag{3}$$

The generalized loading vector l_i is defined as [18]:

$$l_i = \frac{L_i}{(L_k L_k)^{1/2}}; \quad L_i = L_1 \underline{S}_i^1 + L_2 \underline{S}_i^2 + L_3 \underline{S}_i^3 \tag{4}$$

In order to take into account the influence of the loading orientation relative to material axes, a scalar anisotropic parameter η is introduced, which represents the projection of the microstructure tensor $\boldsymbol{a}$ on the current loading direction $\underline{l}$ (3):

$$\eta = a_{ij} l_i l_j = \frac{tr(\boldsymbol{a}\boldsymbol{\Sigma}^2)}{tr(\boldsymbol{\Sigma}^2)} \tag{5}$$

Decomposing the microstructure tensor $\boldsymbol{a}$ into a spherical part $\hat{\eta}$ and deviatoric one A_{ij}, the scalar parameter (5) can be rewritten as:

$$\eta = \hat{\eta}(1 + A_{ij} l_i l_j); \quad \text{with} \quad A_{ij} = (a_{ij} - \hat{\eta}\delta_{ij})/\hat{\eta}; \quad \hat{\eta} = \frac{a_{kk}}{3} \tag{6}$$

A more general expression for η can be obtained by including higher-order tensors:

$$\eta = \hat{\eta}(1 + A_{ij} l_i l_j + A_{ijkl} l_i l_j l_k l_l + A_{ijklmn} l_i l_j l_k l_l l_m l_n + \ldots) \tag{7}$$

As proposed by [18], a specific case of this representation of microstructure is to define the high order tensors via dyadic products of $\boldsymbol{A}$, i.e. $A_{ijkl} = b_1 A_{ij} A_{kl}$; $A_{ijklmn} = b_2 A_{ij} A_{kl} A_{mn}$, etc., so that

$$\eta = \hat{\eta}[1 + A_{ij} l_i l_j + b_1 (A_{ij} l_i l_j)^2 + b_2 (A_{ij} l_i l_j)^3 + b_3 (A_{ij} l_i l_j)^4 + \ldots] \tag{8}$$

With this notion of anisotropy parameter (8), the macroscopic behavior of Tournemire shale can be studied in the framework of elastoplasticity by assuming the yield criterion in the following form:

$$\Phi(\boldsymbol{\Sigma}, f, \rho, \eta) = 0 \tag{9}$$

2.2. Macroscopic criterion of the heterogeneous rock

The effects of anisotropy on the material strength are described by the scalar parameter η, which reflects the orientation dependent. Now, consider the studied rock as a heterogeneous material at both microscopic and mesoscopic scales. The influences of the porosity f in the porous clay matrix and the volume fraction ρ of particles on the macroscopic behavior of the composite will be considered in this subsection.

Considering the pressure-dependency of geomaterials, assume that the solid phase of the clay matrix obeys to a Drucker–Prager type criterion:

$$\phi^m(\tilde{\boldsymbol{\sigma}}) = \tilde{\sigma}_d + T(\tilde{\sigma}_m - h) \le 0 \tag{10}$$

where $\tilde{\boldsymbol{\sigma}}$ denotes the local stress in the solid phase, $\tilde{\sigma}_m = \mathrm{tr}\tilde{\boldsymbol{\sigma}}/3$ being the local mean stress, $\tilde{\sigma}_d = \sqrt{\tilde{\boldsymbol{\sigma}}' : \tilde{\boldsymbol{\sigma}}'}$ the generalized local deviatoric stress with $\tilde{\boldsymbol{\sigma}}' = \tilde{\boldsymbol{\sigma}} - \tilde{\sigma}_m \mathbf{1}$. The symbol "∼" is used in order to make difference between the microscopic stresses of the solid phase and the mesoscopic ones $\boldsymbol{\sigma}$ in the clay matrix. $\boldsymbol{\Sigma}$ is used to denote the macroscopic stresses of the rock composite. The parameter h represents the hydrostatic tensile strength while T the frictional coefficient.

For the porous medium with the Drucker–Prager matrix described above, Maghous et al. [12] have obtained a closed form macroscopic strength criterion by using a non linear homogenization technique based on the modified secant method. Note that this method was initially proposed by [21,22] as a variational method and was later interpreted as "a modified secant method" by [28,29,23]. In their work, in order to consider the non uniform