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Research Paper

Three-dimensional fractal distribution of the number of rock-mass fracture surfaces and its simulation technology

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ABSTRACT

The number of rock-mass fractures obeys a fractal distribution. In this study, numerical simulation was performed to confirm that the number of rock-mass fracture surfaces also obeys a three-dimensional fractal distribution. A simulation model is proposed here for the three-dimensional fractal distribution of the number of rock-mass fracture surfaces. Random distributions of fracture surfaces (RDFS) can be divided into three types: heavy random, weakly random, and distributed by group. By extensive calculations and theoretical deduction, the relation of the three types RDFS between the 3D fractal dimension (D_S) of the fracture surfaces and the fractal parameter (D_L) of the fracture trajectory in a 2D profile, $D_L = D_S - 1$, has been derived. It has further been proved that the 2D fractal dimension (D_L) is not related to the initial value of the fracture-surface distribution. The initial value of the 2D fractal distribution (N_L) shows a linear relation with the initial value of the 3D fractal distribution (N_S), $N_L = kN_S$, where k is determined by the projection relation between the rock-mass profile and the 3D fracture surface.

The program developed in this research to simulate fractal analysis of rock-mass fractures and the correlation between 2D and 3D fractal characteristics could facilitate study of the fracture-trajectory distribution on any rock-mass profile and that of the fracture surfaces in any subblock. This work provides theoretical and technical support for stability analysis in geological engineering.

1. Introduction

Rock blocks and fracture surfaces (structural planes) make up a rock mass. The stability and mechanical properties of a rock mass, such as anisotropy, heterogeneity, permeability, conductibility, bearing capacity, and displacement deformation, are related to the quantity, scale, and distribution pattern of fracture surfaces. Furthermore, fracture surfaces are a key factor in rock-mass quality evaluation. The distribution of rock fissures, cracks, and fractures is an important topic in geology and geological engineering. Many studies have been carried out in this area to investigate the spacing, density, and random distribution of fractures and cracks. Statistical methods or numerical simulations based on statistical and probabilistic methods are commonly used, e.g., [12,9,14,13,8,6]. In recent years, Xu and Dowd [18] describes a software that can be used to generate 2D and 3D fractures networks. Zhang and Ding [19] presents an expression for estimating the standard deviation of estimator, and the expression is used to analyze a set of fracture traces simulated with the FracMan Code and a natural fracture trace network with two orthogonal fracture sets. Toth [16] introduces novel algorithms for determining fracture length distribution and spatial density using 1D datasets. Li et al. [7] estimate the fracture trace length distribution using PWWs and L-moments. Zeeb et al. [4] provide fracture network evaluation program for evaluate large amounts of fracture data. Umili et al. [17] describes a new automatic method for discontinuity traces mapping and sampling on a rock mass digital model. Annavarapu et al. [15] examined the normalized frequencies of overall joint spacing and joint spacing within particular joint sets and used negative exponential and lognormal model to model joint space. Another developing trend is to study the fractal characteristics of fractures. Many researchers have studied the distributions of geologic bodies and their fractures, fissures, and cracks on the basis of their similarities using the principles of fractal geometry and applying fractal geometric methods. These studies have provided a greater understanding of some of these laws of nature [11,21,1,2,10,3] studied the distribution patterns of natural geological cracks and fractures.

Since 1990, [20] systematic research into the distribution and number of fractures has been carried out. Kruhl [5] commented

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on the use of fractal-geometry techniques to quantify the structure of a complex rock mass. He also investigated scaling regimes, inhomogeneity, and anisotropy.

Due to differences in the fracture-trajectory distribution at different positions on the profile, 2D fractal analysis is inadequate to obtain a thorough understanding of the fracture-surface distribution in a rock mass. Some mistaken ideas may even arise from 2D fractal analysis alone. It is of great importance to verify theoretically the fractal pattern of fracture surfaces and to present a rigorous way to describe the 3D fractal geometry of the fracture-surface distribution in a rock mass, as well as the relation between 3D and 2D fractal parameters of fracture surfaces. Development of simulation techniques to represent the fractal geometry of the 3D distribution of fracture surfaces and compilation of the corresponding simulation program constitute the major subjects of this paper.

2. 3D fractal description of the distribution of the number of fracture surfaces in a rock mass

A method for studying the bulk density of fracture surfaces in a rock mass is used here. Based on the basic principles of fractal geometry and the definition of the fractal of the number of 2D fracture trajectories, a method is presented here to describe the 3D fractal of the number of fracture surfaces in a rock mass:

(1) Consider a cube with side length L_0 (L_0 is the initial scale of observation; it can be any value). The number of fracture surfaces $N(L_0)$ with area equal to or greater than L_0^2 in the cube is estimated.
(2) The side length L_0 is bisected to give eight subcubes with side length $\delta_1 = L_0/2$. In each subcube, the number of fracture surfaces with area equal to or greater than δ_1^2 is estimated. The additive operation indicates that the number of fracture surfaces of scale less than $L_0/2$ is $N(L_0/2)$.
(3) By the same method, the total number of fracture surfaces $N(L_0/2^k)$ with area equal to or greater than ${\delta_k}^2$ is obtained in 2^{3k} subcubes with side length $\delta_k = L_0/2^k$.
(4) Using the method described above, the numerical sequence $N(L_0/2^k)$ of the numbers of fracture surfaces at different scales $L_0/2^k$ is obtained.
(5) The result is plotted on the log L-log N plane, yielding a curve in power-function form,

$$N(\delta) = N_s \delta_s^{-D}, \tag{1}$$

where N_S is the initial value of the number of fracture surfaces, which is equal to the number of fracture surfaces with area no less than 1, and D_S is the fractal dimension of the number of fracture surfaces.

3. Simulation model of the 3D fractal distribution of the number of fracture surfaces in a rock mass

3.1. Simulation model of 3D fractal distribution

According to Eq. (1), the number of fracture surfaces at scale δk is $N(\delta k)$. To construct a simulation model for the 3D fractal distribution of fracture surfaces in a rock mass, the following hypotheses are introduced: (1) the fracture surfaces in the rock mass are disk-shaped spatial planes; (2) the line of intersection between the fracture surface and the observation plane in the rock mass is the fracture trajectory; (3) the fracture surfaces are randomly distributed in the rock mass. Based on these hypotheses, the following controlling parameters are introduced: (1) the number of groups of fracture surfaces (g); (2) the initial scale of measurement (L_0); (3) the 3D fractal dimension (D_S); (4) the initial number of fracture surfaces (N_S); and (5) the normal vector to the fracture surface, $\mathbf{n_r}(l, m, n)$.

Then the size and position of any fracture surface in the rock mass can be uniquely defined by the position coordinates of the center point of fracture surface $a(x_c, y_c, z_c)$, the radius r, and the normal vector $\mathbf{n}(l, m, n)$. Moreover,

$$\begin{aligned} & l(x-x_c) + m(y-y_c) + n(z-z_c) = 0 \\ & (x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 \leqslant r^2 \\ & r = \sqrt{\tfrac{1}{\pi}}\delta_k \end{aligned}, \tag{2}$$

where r is the radius of the disk as calculated using the areal equivalence of the disk and the rectangle.

The coordinate system is determined as shown in Fig. 1. Both the dip angle ST of the fracture surface and the strike angle SP range from 0° to 180°. When the strike angle has the same direction as the negative side of the x-axis, it is an acute angle, otherwise it is an obtuse angle; when the dip angle has the same direction as the negative side of the x-axis on the XOZ plane defined by the dip direction of the fracture surface, it is an acute angle, otherwise it is an obtuse angle. The normal vector $\vec{n}(l, m, n)$ can be calculated using the dip angle ST and the strike angle SP as,

$$\begin{aligned} & l = \sin(ST)\sin(SP) \\ & m = \cos(ST) \\ & n = -\sin(ST)\cos(SP) \end{aligned}. \tag{3}$$

According to this theory, the fracture distribution of any profile in the rock mass can be determined either by Eqs. (2) and (3) or by the profile equation ($x = x_m$, $y = y_m$, $z = z_m$).

For example, when $x = x_m$, a monadic quadratic equation in terms of z can be obtained by combining Eqs. (2) and (3):

$$\begin{aligned} & Az^2 + Bz + c = 0 \\ & A = 1 + n^2/m^2 \\ & B = 2 \cdot l \cdot n(x_m - x_c)/m^2 \\ & C = 1 + l^2/m^2 - r^2 \end{aligned}. \tag{4}$$

The root calculated by Eq. (4) is the z-coordinate of the endpoint of the line of intersection between the fracture surface and the profile. When the equation has only one root, this means that the fracture surface and the profile have only one intersection point. When the equation has no solution, the fracture surface does not intersect with the profile.

Based on the theory of 3D fractals, when the fractal-geometry parameters of fractures in the rock mass are already known (by outcrop measurement and core drilling), a simulation model of

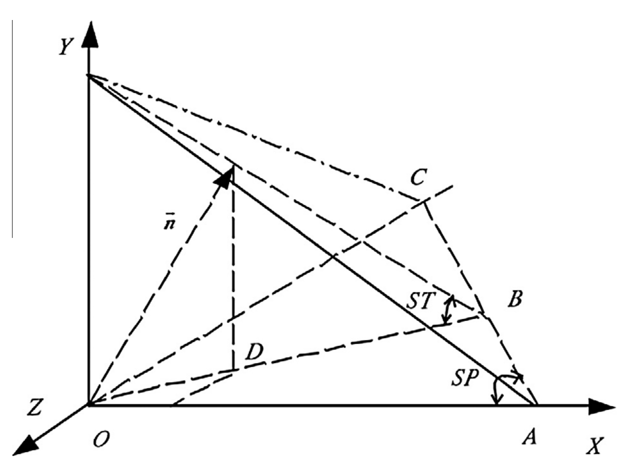

Fig. 1. Relation of dip angle, azimuth and normal vector.