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Research Paper

Methods to project plasticity models onto the contact surface applied to soil structure interactions

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ABSTRACT

In this work two new concepts for a direct application of plasticity models within a frictional contact description are developed. These concepts can be used in conjunction with all different kinds of contact formulations and solution methods. Additionally, all types of plasticity models can be projected onto the contact surface. The advantage of these concepts is shown exemplary in the modeling process of soil-structure interactions where the projected plasticity models are able to describe the soil behavior at the contact surface. The numerical implementation of the new frictional relations is based on the Mortar method. A new type of mixed formulation is also introduced combining the augmented Lagrangian method to enforce the normal contact constraint with the penalty regularization written in Hellinger–Reissner form to implement the tangential contact behavior. This reformulation leads to a reduction of the CPU time compared to the standard penalty regularization, if the Mortar method is used. Finally, the numerical investigation of a direct shear test shows the accurate reproduction of the typical stress–strain relation of the soil at the contact surface.

1. Introduction

When investigating simulations of sliding contacts, Coulomb's law is mostly preferred to model the frictional behavior. Even within the highly complex modeling process of soil-structure interactions Coulomb's law is applied [1,2], although the simulation of a pile penetration process shows a large difference between numerical and experimental results (Fig. 1, [3]). As a consequence new frictional models where developed in [3] improving the slip behavior. Unfortunately, a large number of additional material parameters was introduced there which have to be determined for each individual contact pair.

Within geotechnical installation processes for piles, anchors or sheet pile walls, mostly the surface of the structure has to be viewed as rough. Experimental measurements of a direct shear test between soil and concrete show that for a rough surface of the structure the response behavior is almost equal to the same test case between two soil specimens [4–6]. This causes the assumption that for these soil-structure interactions the real contact zone lies completely within the soil (Fig. 2). Since many soil models are able to represent the 3-dimensional geomechanical behavior exactly, the description of the mechanics at the contact layer can be improved by the use of such models. Until now either interface elements [7,8], or special joint elements [9,10] are used to model the contact interface by use of soil models. Additionally, some interface models exists where the rough surface structure is taken into account [11,12]. These models are limited to small sliding and only an incorporation into contact formulations makes it possible to simulate more realistic situations where large relative movements occur, like pile installation processes for instance. Hence within this work two different strategies are developed each able to incorporate the plasticity models directly into a friction model. Another advantage of this projection schemes is that no additional parameters are needed.

A big challenge of any projection method, especially for soil-structure interactions, is the correct reproduction of the dilatant or contractant behavior at the contact surface. A direct integration of these influences into a contact model would lead in the case of contractancy to a penetration of one body into the other which is not allowed or in the case of dilatancy to a release of the contact during the sliding process which is not reasonable. Additionally, yield criteria are often formulated in terms of three stress invariants whereas slip laws are mostly based on the norm of the tangential stress vector and on the absolute value of the normal pressure. Hence a direct link between these invariants is not possible. However in the literature some relation between contact and

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continuum are disclosed. For instance, the 3-dimensional Mohr–Coulomb yield criterion is the natural extension of the two dimensional Coulomb slip rule [13]. Using the penalty regularization for the tangential contact a formulation analog to the elasto-plastic theory can be exploited in the modeling process [14–16]. Both relations are providing the basis for the developed projection methods.

If the surface of the structure can be assumed as perfectly smooth, contact takes place directly at the interface of soil and structure and Coulomb's law can be used, as can be seen in the outcomes of experimental tests between steel and soil in [17,18]. Only a proper coefficient of friction has to be determined.

For the numerical implementation of the new friction models first the boundary value problem has to be described. The discretization of the leading equations within a finite element framework is displayed in Section 2 where the focus lies especially on the contact part. In this work the Mortar method [19] is used exemplary for the discretization of the contact part leading to a robust solution algorithm [20]. A new type of mixed version is embedded in the Mortar framework which combines the augmented Lagrangian method [21,22] for the normal contact description with the penalty regularization given in Hellinger–Reissner form for the tangential part delivering a stable solution technique for contact models.

A soil model based on the framework of the elasto-plastic theory which is able to include the porous structure of the soil [23,24] is stated in Section 3. Additionally, two regularization schemes are mentioned shortly at the end of this section which stabilize the back-projection within the return mapping algorithm and avoid oscillations between the elastic and plastic state of a material point.

Sections 4 and 5 describe the two developed projection methods in detail. The first one transforms the plasticity equations properly into frictional formulations using the connection between Coulomb slip rule and Mohr–Coulomb yield criterion. The second concept integrates the plasticity model directly into the slip rule formulating a continuum stress dependent coefficient of friction and normal contact force. The results of the new projection concepts are shown at the end of each section within numerical investigations of a direct shear test. There the outcomes are compared with the results of a corresponding 3-dimensional setup using interface elements in between of the two contact specimens. The presented work is closed with an evaluation of both projection schemes in Section 6.

2. A mixed Mortar method

The new friction models can be solved with all kinds of contact formulations. In this work the new contact equations are included into a solution method that is embedded in a Mortar framework. To have a natural transformation from plasticity to friction and additionally a strong enforcement of the non penetration condition

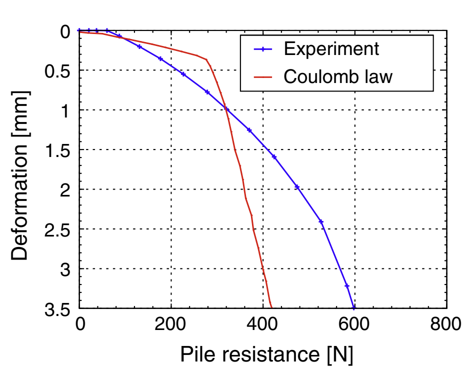

Fig. 1. Comparison of experimental and numerical results of a pile penetration test [3].

a new type of mixed formulation is proposed. There the normal contact constraint is solved using the augmented Lagrangian method and the tangential constraint is regularized with the penalty method written in Hellinger–Reissner form.

2.1. Boundary value problem

In the following investigations only quasi-static cases will be considered that rely on constitutive models for small strain applications. Additionally, the influence of the gravity force is neglected to concentrate on the pressure dependency of the numerical response behavior. The balance equation of momentum for each contacting body simplifies then to the requirement that the divergence of each stress $\boldsymbol{\sigma}^i$ has to be zero

$$\operatorname{div} \boldsymbol{\sigma}^i = 0 \quad \text{in} \quad B^i. \tag{1}$$

Here index ($i = 1$) stands for the body which surface will be denoted as slave surface and ($i = 2$) denotes the master surface. This distinction was introduced in [25]. The boundary of each body is subdivided into the Neumann boundary where the applied traction $\bar{\mathrm{t}}^i$ is given and into the Dirichlet boundary where the applied displacements $\bar{\mathrm{u}}^i$ are prescribed

$$\begin{aligned} \boldsymbol{\sigma}^i \mathrm{n}^i &= \bar{\mathrm{t}}^i \quad \text{on} \quad \partial_\sigma B^i \\ \mathrm{u}^i &= \bar{\mathrm{u}}^i \quad \text{on} \quad \partial_u B^i. \end{aligned} \tag{2}$$

In the case of contact a third boundary part $\partial_c B$ has to be considered which denotes the contact area. Hence the boundary of each body is uniquely subdivided in three different regions $\partial_\sigma B^i \cap \partial_u B^i \cap \partial_c B = \emptyset$. On the contact boundary the normal gap

$$g_\mathrm{N} = \left(\mathrm{x}^2 - \mathrm{x}^1\right) \cdot \mathrm{n}^1 \tag{3}$$

and the pressure λ_N determines the contact behavior in normal direction. For the computation of g_N the actual position vectors x^i of the master and of the slave surface are used, see also [16]. Contact takes place, if the normal penetration is equal to zero. In the case of non touching bodies the contact pressure has to vanish leading to the set of inequalities which can also be written in the Karush–Kuhn–Tucker form

$$g_\mathrm{N} \geqslant 0, \quad \lambda_\mathrm{N} \leqslant 0, \quad g_\mathrm{N}\lambda_\mathrm{N} = 0 \quad \partial_c B. \tag{4}$$

Similarly, for the tangential contact two inequalities can be stated to define the stick or the slip state of the surface point. Thereby friction takes place, if the slip rule f^c is equal to zero introducing additionally a slip rate ($\dot{\gamma} > 0$). In analogy to the elasto-plastic theory, an evolution equation for the tangential gap g_T is defined where the direction of sliding corresponds to the direction of the tangential contact stress vector $\boldsymbol{\lambda}_\mathrm{T}$

$$\left.\begin{aligned} \dot{\gamma} \geqslant 0, \quad f^c(\|\boldsymbol{\lambda}_\mathrm{T}\|, \lambda_\mathrm{N}) \leqslant 0, \quad \dot{\gamma} f^c = 0 \\ \dot{\mathrm{g}}_\mathrm{T} = \dot{\gamma}\frac{\boldsymbol{\lambda}_\mathrm{T}}{\|\boldsymbol{\lambda}_\mathrm{T}\|}. \end{aligned}\right\} \quad \text{on} \quad \partial_c B \tag{5}$$

A detailed derivation and explanation of the mentioned equations can be found for instance in standard contact textbooks [16,26].

2.2. Finite element discretization

For the solution of contact problems the finite element method is often employed. For this, first the balance of momentum (1) of each body together with the normal (4) and tangential contact constraints (5) has to be written in a weak form

$$\sum_{i=1}^{2} G^i(\mathrm{u}, \boldsymbol{\eta}) + G_u^c(\mathrm{u}, \boldsymbol{\eta}, \boldsymbol{\lambda}) + G_l^c(\mathrm{u}, \boldsymbol{\eta}, \delta\boldsymbol{\lambda}) = 0. \tag{6}$$

The formulation and discretization of the virtual work part of the two contacting bodies $G^i(\mathrm{u}, \boldsymbol{\eta})$ can be found in standard finite