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Research Paper

Importance sampling based algorithm for efficient reliability analysis of axially loaded piles

Haijian Fan [a,*], Robert Liang [b]

[a] E.L. Robinson Engineering, 1801 Watermark Drive, Suite 310, Columbus, OH 43215, United States
[b] Department of Civil Engineering, The University of Akron, 244 Sumner Street, ASEC 210, Akron, OH 44325, United States

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ABSTRACT

In reliability analysis, the crude Monte Carlo method is known to be computationally demanding. To improve computational efficiency, this paper presents an importance sampling based algorithm that can be applied to conduct efficient reliability evaluation for axially loaded piles. The spatial variability of soil properties along the pile length is considered by random field modeling, in which a mean, a variance, and a correlation length are used to statistically characterize a random field. The local averaging subdivision technique is employed to generate random fields. In each realization, the random fields are used as inputs to the well-established load transfer method to evaluate the load–displacement behavior of an axially loaded pile. Failure is defined as the event where the vertical movement at the pile top exceeds the allowable displacement. By sampling more heavily from the region of interest and then scaling the indicator function back by a ratio of probability densities, a faster rate of convergence can be achieved in the proposed importance sampling algorithm while maintaining the same accuracy as in the crude Monte Carlo method. Two examples are given to demonstrate the accuracy and the efficiency of the proposed method. It is shown that the estimate based on the proposed importance sampling method is unbiased. Furthermore, the size of samples can be greatly reduced in the developed method.

1. Introduction

Monte Carlo simulation (MCS) is widely used for reliability analysis because of its mathematical simplicity and robustness. An important application of MCS is to evaluate the probability of failure

$$P_f = P(G \leqslant 0) = \int_{G\leqslant 0} f(\boldsymbol{x})d\boldsymbol{x} = \int I[G \leqslant 0] f(\boldsymbol{x})d\boldsymbol{x} \tag{1}$$

where $P(\cdot)$ denotes a probability measure, $f(\boldsymbol{x})$ denotes the joint probability density function (PDF) of random vector $\boldsymbol{x}$, $I[\cdot]$ denotes an indicator function, G denotes a limit state function (LSF), and failure is denoted as $G \leqslant 0$. A general definition of the limit state function is written as

$$G(\boldsymbol{x}) = C - D(\boldsymbol{x}) \tag{2}$$

where C and D denote the "capacity" and the "demand" in a broad sense. With the probability of failure, the reliability index β can be evaluated accordingly

$$\beta = \Phi^{-1}(1 - P_f) = -\Phi^{-1}(P_f) \tag{3}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of the cumulative distribution function for the standard normal variable.

A common problem with the evaluation of the failure probability is that the number of dimensions in reliability problems may be large, making it difficult to evaluate the numerical integration (i.e., Eq. (1)) directly. In that case, the integration can be evaluated as the expectation of the indicator function using Monte Carlo integration [15]. Simply speaking, the probability of failure P_f is approximated as the ratio of the number of failure events to the total number of samples

$$P_{f,\mathrm{MCS}} = \frac{1}{n}\sum_{i=1}^{n} I_i[G \leqslant 0] \tag{4}$$

where n is the sample size, the subscript "MCS" indicates that the estimate is evaluated using the crude MCS method. The indicator function is equal to 1.0 if $G \leqslant 0$, and it is zero, otherwise. Note that in the crude MCS method, the random variables $\boldsymbol{x}$ are drawn from the joint PDF f.

To guarantee the convergence of Eq. (4), the sample size n has to be sufficiently large. According to Ang and Tang [2], the degree of

* Corresponding author.
E-mail addresses: fan613@gmail.com (H. Fan), rliang@uakron.edu (R. Liang).

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precision for the estimate of Eq. (4) can be measured by its coefficient of variation (COV) δ_p

$$\delta_p = \sqrt{\frac{1 - P_{f,\mathrm{MCS}}}{n \cdot P_{f,\mathrm{MCS}}}} \tag{5}$$

Eq. (5) indicates that the coefficient of variation is affected by the sample size n and P_f. Based on Eq. (5), a back-calculation can be used to estimate the sample size n as follows

$$n = \frac{1 - P_{f,\mathrm{MCS}}}{\delta_p^2 \cdot P_{f,\mathrm{MCS}}} \tag{6}$$

Based on Eq. (6), it is concluded that the sample size n is related to P_f and its COV δ_p. The smaller δ_p and P_f are, the larger n is. The probability of failure P_f is usually a small number. Thus, the sample size n is usually large such that the convergence of Eq. (4) is guaranteed. According to Robert and Casella [15], a rule of thumb for selecting a sample size is that n is approximately ten times the reciprocal of the probability level, if the COV for P_f is taken as 30%. For example, the sample size is at least 10,000 if $P_f = 1/1000$. A larger sample size is warranted if a smaller δ_p is desired. Based on the above discussion, it is well understood that the crude Monte Carlo statistical methods are usually computationally expensive as a result of repetitive evaluation of the indicator function. Particularly, it becomes more demanding if the evaluation of the indicator functions is complex.

In the reliability analysis for axially loaded piles, the authors previously developed a performance based design approach using the crude MCS method [9]. To improve the computational efficiency of the Monte Carlo method, an importance sampling (IS) based algorithm for fast reliability evaluation on axially loaded piles was developed [10]. In the proposed algorithm, the instrumental function for importance sampling is constructed by shifting the original PDF such that the mean is at the point having the maximum probability density of the failure surface. By sampling more heavily from the region of interest and then scaling the indicator function back by a ratio of probability densities, a faster rate of convergence can be achieved while maintaining the accuracy of the estimate. Although the numerical theories of the algorithm has been published, this paper has the following enhancements:

(1) A discussion of the load transfer model [5] for axially loaded piles was added. The load transfer model is employed to evaluate the load–displacement relationship, which is used to determined whether failure occurs. If the displacement at the pile top exceeds the allowable limit, failure is said to occur. Because the proposed algorithm is implemented by a computer code to calculate the failure probability, it is essential to include a brief description of the load transfer model to explain the algorithm.
(2) Soil properties are statistically modeled as lognormal variables. The spatial variability of soil properties is considered by random field modeling. The random fields for soil properties are simulated by using the local averaging subdivision (LAS) method [11].
(3) Two examples – one for homogeneous clay sites and the other for non-uniform clay sites – are presented to illustrate the accuracy and efficiency of the developed importance sampling method. The first example has been expanded to discuss the influences of soil spatial variability on the reliability evaluation. Moreover, the second example was added to contrast the computational efficiencies of the crude Monte Carlo method and the proposed numerical algorithm.

2. Load transfer model

The analysis of axially loaded piles is a nonlinear soil-pile interaction problem that is solved by iterative numerical algorithms. Numerous methods are available to analyze the response of an axially loaded pile, such as the finite element method and the load transfer method. The load transfer method (e.g., [5] is widely used because of its accuracy and simplicity. In the load transfer method, the soil reaction to axial loads is modeled by nonlinear t–z curves and q–w curves, where t represents side friction, z represents the vertical movement of the pile, q represents the end bearing, and w represents the vertical movement at the pile tip. A schematic diagram of the load transfer model is shown in Fig. 1.

3. Random field modeling

Soil properties such as undrained shear strength and friction angle are needed as input to construct the load transfer curves (t–z curves and q–w curves). The soil properties are uncertain due to intrinsic variability, measurement errors, and interpretation errors. The variations of soil properties can directly affect the t–z curves and q–w curves in the load transfer model. Consequently, these variations can exert significant influence on the calculated load–displacement curve. Therefore, the modeling of soil variability is of great importance in reliability assessment.

Two statistical parameters, namely mean μ and variance σ^2, are required to characterize the variability of a soil property at the point level. The mean measures the center of a dataset while the variance measures the dispersion from the mean. One commonly used probability distribution for soil properties is the lognormal distribution (e.g., [12]. The use of lognormal distribution ensures that soil properties are always non-negative. The PDF for lognormal distribution is given as follows

$$f(x|\mu_{\ln x}, \sigma_{\ln x}) = \frac{1}{x\sigma_{\ln x}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\frac{(\ln x - \mu_{\ln x})^2}{\sigma_{\ln x}^2}\right] \tag{7}$$

where $\mu_{\ln x}$ and $\sigma_{\ln x}$ are the distribution parameters. The distribution parameters can be determined based on the mean μ_x and the standard deviation σ_x

$$\sigma_{\ln x} = \sqrt{\ln\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right)} \tag{8}$$

$$\mu_{\ln x} = \ln \mu_x - 0.5\sigma_{\ln x}^2 \tag{9}$$

By taking the logarithm, a lognormally distributed random variable can be transformed to a normal variable with mean $\mu_{\ln x}$ and standard deviation $\sigma_{\ln x}$. For mathematical simplicity, it is preferable to have normal variables. The advantage of such a transformation will become clear in the subsequent discussion. It should be noted that other probability distributions such as normal distribution are possible choice for modeling soil properties.

In addition to the mean and the variance, a third parameter called correlation length θ is required to characterize the spatial variability of a random variable [16]. The correlation length is needed to define a correlation function, which describes how random variables are correlated at different separation distances. For example, the correlation function for Markov process is given below

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \tag{10}$$

where $\rho(\tau)$ is the correlation coefficient at the separation distance of τ. Eq. (10) says the correlation coefficient decays exponentially