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Large deformation elastic electro-osmosis consolidation of clays

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ABSTRACT

This paper presents the theoretical background of an elastic electro-osmosis consolidation model for saturated soils experiencing large strains, which considers volumetric strains induced by changes in both the hydraulic and electric driven pore water flows. Three fully coupled governing equations, considering the soil mechanical behaviour, pore water transport and electrical field, and their numerical implementation within an updated Lagrangian finite element formulation, are presented. The proposed model is first verified against a classical one-dimensional analytical solution for electro-osmosis consolidation to demonstrate its accuracy and efficiency. Then, various numerical examples are investigated to study the deformation characteristics and time dependent evolution of excess pore pressure. Finally, the importance of considering large strains in a consistent and proper way is demonstrated, and differences compared to models based on small strain theory are highlighted.

1. Introduction

In a porous medium, electro-osmosis is the water flow from the positive electrode to the negative electrode when an electrical gradient is applied. Geotechnical and geo-environmental engineers have been interested in electro-osmosis for many years as a method of soil improvement, which includes electro-osmotic dewatering, ion injection, contaminant removal, electro-bioremediation and electro-chemical remediation. In particular, electro-osmosis as a method of ground improvement for soft soil is receiving much attention. When traditional ground improvement techniques, such as surcharge pre-loading, vertical drains and vacuum pre-loading are not appropriate for a particular situation, innovative techniques such as electro-osmosis need to be considered.

In 1939 Casagrande introduced the treatment of soil by electro-osmosis to improve its engineering properties. Since then, electro-osmosis has been successfully applied in various fields as an economical and time saving method, for example to improve friction pile capacity [1,2], to control the pore water at excavation sites [3], to stabilize excessive foundation deformations [4], for the strengthening and stabilization of soft clays [5–7], for the consolidation of marine sediments for land reclamations [8], and for the dewatering of mine tailings [9–11]. At the same time, numerous laboratory studies have been conducted to understand the suitability of its application, the anticipated effects of the treatment, its efficiency and effectiveness, and different treatment costs. These papers include studies of the optimal voltage gradient and current density [12–14], the effects of polarity reversal and current intermittence [10,15–17], chemical treatment to improve electro-osmosis consolidation [7,18–21], different electrode materials [11,17,22–24] and the effect of zeta potential [25–27].

The theory of one dimensional (1D) electro-osmosis consolidation was first developed by Esrig [28] and, based on Esrig's equation, Wan and Mitchell [16] presented an analytical solution for 1D preloading. Feldkamp and Belhomme [29] later derived an analytical solution for large strain 1D electro-osmosis consolidation. Lewis and Garner [30] presented a two dimensional (2D) finite element solution for modelling the coupling effect of the electric and hydraulic gradients. Shang [31,32] developed a 2D analytical model combining pre-loading and the electro-osmosis consolidation of clay soils. Rittirong and Shang [33] presented a 2D finite difference model to analyse indirectly the subsurface settlement and undrained shear strength. Iwata and Jami [34] presented a numerical model to simulate the combined electro-osmosis dewatering and mechanical response, using the Terzaghi–Voigt combined model to consider creep deformation. Yuan et al. [35] proposed a multi-dimensional finite element model to consider the coupled process of mechanical behaviour, hydraulic flow and electrical flow during electro-osmosis consolidation. Hu et al. [36] developed a physics-based model for electro-osmosis consolidation, coupling displacement, pore water flow and electrical field, in which the nonlinear variation of the mechanical and electro-osmosis properties of the soil mass were included.

In geotechnical consolidation problems, the deformation is usually coupled with the flow of pore fluids based on the consolidation theory of Biot [37]. Carter et al. [38,39] presented a model for large deformation elastic and elastoplastic consolidation for the first

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time, whereas Prevost [40,41] proposed a generalized incremental form of large deformation consolidation involving material nonlinearity under both static and dynamic loadings. Zienkiewicz and Shiomi [42] and Meroi et al. [43] derived a numerical model of dynamic large deformation consolidation in saturated and unsaturated porous media. Borja et al. [44,45] developed a mathematical model for large deformation elastoplastic consolidation of fully saturated soil media and then implemented it into a finite element program. Li et al. [46] presented a dynamic hyperelastic consolidation model under finite strain. Andrade and Borja [47] proposed an elastoplastic large deformation model for partially saturated consolidation. Nazem et al. [48] presented an arbitrary Lagrangian–Eulerian method for analysing large deformation elastoplastic consolidation problems.

The electro-osmosis consolidation of soft clay is a coupled process involving mechanical behaviour, hydraulic flow and electrical flow under large deformation. The conceptual basics and theory of electro-osmosis consolidation have been studied by many of the above authors, but few analytical and numerical studies have considered the fully coupled process of soil mechanical behaviour, pore water flow and electrical flow. Furthermore, the geometric nonlinearity of the solid skeleton has not been considered in the numerical modelling of electro-osmosis consolidation, although much of the previous experimental work mentioned above reported large scale deformations during tests.

This contribution develops a numerical model for the finite element solution of hydro-mechanical–electrical processes in fully saturated porous media with isotropic elastic soil skeleton behaviour at large strains. Three fully coupled governing equations considering force equilibrium, pore water transport and electrical current flow are presented and solved, via the finite element method in the space domain and an implicit integration scheme in the time domain, and an updated Lagrangian (UL) method is employed to solve for large deformations.

The paper is organized as follows. In Section 2 the governing equations of electro-osmosis consolidation, as well as the kinematics and deformations for updated Lagrangian formulations, are recalled. Furthermore, the Jaumann stress rates used in large deformation analysis, which consider the effect of rigid body rotations, are discussed. Then the linearization of weak formulations of the equilibrium condition is briefly introduced. Section 3 addresses the finite element procedure in detail, and the discretization of the governing equations in space and time is described. Thus a classical finite element formulation is obtained, comprising increments of displacements, pore water pressure and electrical potential as the primary variables. The nonlinear finite element equations are solved using the modified Newton–Raphson scheme and the numerical algorithm has been implemented into a finite element code. In Section 4, the proposed methodology is first validated against the analytical solution developed by Esrig [28]. This is followed by a series of numerical examples, to investigate both the performance of large strain electro-osmosis consolidation, as well as the differences between small and large strain solutions.

2. Theoretical and governing formulations

In this paper, an isotropic fully saturated soil with an incompressible pore liquid and soil particles is considered. The governing equations for the equilibrium of force, electric potential and hydraulic head are derived based on the following assumptions: the temperature in the soil is constant during the simulation; the effect of electrical–chemical reaction is negligible; the current due to electrophoresis of the fine grained particles is negligible [28]; the flow of fluid due to the electrical and hydraulic gradients may be superimposed to obtain the total flow [28]; Ohm's law is valid; Darcy's law is valid; and the electrical gradient caused by the movement of ions is negligible compared to the applied electrical field.

2.1. Kinematics and deformations

To deal with large deformation problems, some assumptions and different configurations need to be discussed first. If the initial (i.e. reference) configuration of a physical body is denoted by Ω^0, an arbitrary point in the body is often represented by its initial coordinates X_i. Let Ω denote the current configuration of the body and x_i represent the current coordinates of the arbitrary point. The mapping function φ is a key relationship between Ω^0 and Ω, which relates the initial and current position vectors. Hence, for a typical time-step, the updated configuration of the body at time $t+\Delta t$ may be written as a function of the configuration at time t and the incremental displacement during the time-step Δt, i.e. [49]

$$x_i^{t+\Delta t} = \varphi_i(X_i, t+\Delta t) \tag{1}$$

The current and fixed reference configurations are related to each other by the displacement, so that an updated position vector can be written as

$$x_i^{t+\Delta t} = X_i + u_i = x_i^t + \Delta u_i \tag{2}$$

whereas the increments in the displacement over the time steps are given by

$$\Delta u_i = u_i^{t+\Delta t} - u_i^t = x_i^{t+\Delta t} - x_i^t \tag{3}$$

During the motion of a body, its volume, surface area, stresses and strains are changing continuously. A fundamental measure of the deformation is given by the deformation gradient, defined as

$$F_{ij} = \frac{\partial \varphi_i}{\partial X_j} = \frac{\partial x_i}{\partial X_j} = \delta_{ij} + \frac{\partial u_i}{\partial X_j} \tag{4}$$

where δ_{ij} is the Kronecker delta and the incremental deformation map is described as $\varphi_i^{t+\Delta t} = \varphi_i^t + \Delta u_i$. Consequently, the deformation gradient at time $t+\Delta t$ can be obtained as $F_{ij}^{t+\Delta t} = f_{ij}^t F_{ij}^t$, where F_{ij}^t is the deformation gradient at time t, and f_{ij}^t is the incremental deformation gradient from time t to $t+\Delta t$, defined as

$$f_{ij}^t = \frac{\partial x_i^{t+\Delta t}}{\partial x_j^t} = \delta_{ij} + \frac{\partial \Delta u_i}{\partial x_j^t} \tag{5}$$

The volume change between the reference and current configurations can be established as

$$d\Omega^{t+\Delta t} = \det F^{t+\Delta t} d\Omega^0 = J^{t+\Delta t} d\Omega^0 \tag{6}$$

where J is the Jacobian matrix which is the determinant of the deformation gradient F. The Green strain tensor is given as [49,50]

$$E_{ij} = \frac{1}{2}(F_{ik}F_{jk} - \delta_{ij}) = -\frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i} \cdot u_{k,j}) \tag{7}$$

With regard to the fluid phases, it is common to assume the motion of the solid as a reference and to describe the fluid relative to the solid. Hence, the velocity of the pore water and electrical particles can be written with reference to the current configuration of the solid body [43,50]. When doing this, some care needs to be taken regarding changes in the permeability tensors due to rigid body rotation [38,39,48], when the material is anisotropic.

By employing the expressions above, the quantities of interest can be transferred to a known configuration where the governing equations can be solved.

2.2. Mechanical equilibrium and effective stress concept

The stress equilibrium equation, at time $t+\Delta t$, can be expressed as