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Laterally loaded rigid piles with rotational constraints

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ABSTRACT

Elastic–plastic, closed-form solutions were developed recently by the author, to capture the nonlinear response of laterally loaded rigid piles. Presented in compact form, the solutions are convenient to use, and sufficiently accurate despite using only two input parameters of the net limiting force per unit length p_u along the pile, and a subgrade modulus k. Nevertheless, piles may be subjected to limited cap-restraints or loading below ground surface, which alter the response remarkably.

This paper provides explicit expressions for estimating loading capacity of anchored piles and develops new solutions for lateral piles with cap-rotation by stipulating a constant p_u or a linear increasing p_u (Gibson p_u) with depth. Lateral loading capacity H_o (at the tip-yield state and yield at rotation point state) and maximum bending moment M_m (at the tip-yield state) are presented against loading locations, and in form of the lateral capacity $H_o - M_o$ (applied moment) locus. The capacity is consistent with available solutions for anchored piles, and caissons with either p_u profile, allowing a united approach from lateral piles to anchored piles. The new solutions are also presented in charts to highlight the impact of rotational stiffness of pile-cap on nonlinear response, offering a united approach for free-head piles through fixed-head piles.

Several advantages of the solutions are identified against the prevalent p–y curve based approach. To estimate the key parameter p_u, values of the resistance factor N_p (=ratio of pile–soil limiting resistance over the undrained shear strength s_u) are deduced using the current expressions against available normalised pile capacity involving the impact of gapping (between pile and soil), pile movement mode, pile slenderness ratio, inclined loading angle (anchored piles) and batter angles (lateral piles). The N_p is characterised by: (i) An increase from 5.6–8.6 to 10.14–11.6, as gapping is eliminated around lateral piles and caissons, and from 1.0–6.1 to 2.8–9.8, as translation is converted into rotation mode of footings. (ii) Similar variations with slenderness ratio between anchors and caissons (without gapping), and among anchors, caissons and pipelines (with gapping). And (iii) A reduction with *loading angles* (anchors) resembling that with batter angles (piles).

1. Introduction

Offshore exploration has propelled analytical, numerical and experimental investigation into bearing capacity of anchored piles [1–6]. This is seen in development of a complicated strength mobilisation (SM) method [7], finite element method (FEM) [8] and plastic limit analysis (PLA) [9], among many others. The study provides the evolution law of the capacity with depth of loading attachment e^- for a constant p_u with depth (p_u = net force per unit length along the pile [FL^{-1}]) and a linearly increase p_u (Gibson p_u). [Note the symbol e is taken as negative (e^-) for depth of attachment to distinguish it from the positive (e^+) loading above ground level]. The FEM and PLA analyses also reveal the variational law of the capacity with loading inclination angle (against horizon). To conduct practical design via these methods, one needs to determine the p_u that should incorporate the combined interaction among pile-movement (translation or rotation) mode, gapping and loading angle. This can be difficult and may be further complicated by ~4 times reduction in the p_u from free-head to fixed-head conditions [10–12]. A realistic p_u may be deduced by fitting available numerical and test results using closed-form solutions, as is evident in the deduced p_u profiles for 52 laterally loaded, flexible piles [13]. The corresponding closed-form solutions for lateral piles with rotating pile-cap, however, are not available. The impact of rotational constraints on the piles by the depth of attachment and/or cap-restraint remains to be determined.

The rigid piles refer to those with a pile–soil relative stiffness E_p/G_s being higher than $0.8322(l/d)^4$ [14]. (Note: E_p is Young's modulus of an equivalent solid pile [FL^{-2}], G_s is soil shear modulus [FL^{-2}], d is an outside diameter of a cylindrical pile [L], and l is the pile embedded length [L]). It should be cautioned that in the use of rigid-pile solutions to predict response of flexible piles [15], bending failure needs to be assessed against maximum

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Nomenclature

A_r	gradient of the limiting p_u [FL^{-2-n}]
$d(r_o)$	outside diameter (radius) of a cylindrical pile (caisson) [L]
E_p	Young's modulus of an equivalent solid cylinder pile [FL^{-2}]
e	eccentricity (free-length) [L]
e^-	depth of attachment for anchored piles
e^+	eccentricity (free-length), i.e. the height from the loading location to the mudline; or $e = M_o/H_t$ for lateral piles [L]
$\bar{e}$	e/l, normalised eccentricity
$G_s, \tilde{G}_s$	shear modulus of the soil, and average of the G_s [FL^{-2}]
H_o	lateral capacity, which is the load H_t at tip-yield or YRP state [F]
$\overline{H}_o$	$H_o/(A_r d l^{1+n})$, normalised capacity
H_t	lateral load applied at an eccentricity of 'e' above mudline [F]
k	modulus of subgrade reaction [FL^{-3}]
k_o	gradient of the modulus of subgrade reaction [FL^{-3-n}], $k_o = k$ at $n = 0$
k_r	rotational stiffness of a pile-cap [FL/rad]
l	embedded pile (caisson) length [L]
LFP	net limiting force profile per unit length [FL^{-1}]
M_m	M_{max}, maximum bending moment within a pile [FL]
$\overline{M}_m$	$M_m/(A_r d l^{2+n})$, normalised maximum bending moment
M_o	bending moment at the mudline level for free-head piles; or restraining bending moment for semi-fixed-head piles [FL]
$\overline{M}_o$	$M_o/(A_r d l^{2+n})$, normalised applied bending moment (at ground level)
n	power to the depth z, $n = 0$ for constant k or p_u, $n = 1$ for Gibson k or p_u
N_p	a resistance factor for limiting force per unit length
$N_{p\alpha}$	a resistance factor for limiting force per unit length under an inclined loading at an angle of α (anchored piles), or a batter angle α (lateral piles)
p, p_u	force per unit length, and limiting value of the p [FL^{-1}]
s_u	undrained shear strength [FL^{-2}]
u	lateral displacement of a rigid pile [L]
u^*	local threshold u^* above which pile–soil relative slip is initiated [L]
u_g, u_t	lateral displacement at mudline level, and pile-head level, respectively [L]
YRP	yield at rotation point
z	depth measured from the mudline [L]
z_m	depth of maximum bending moment [L]
$z_o(z_1)$	slip depth initiated from mudline (pile-base) [L]
z_r	depth of rotation point [L]
$\bar{z}_r$	z_r/l, normalised depth of rotation point
z_*	slip depth at tip-yield state [L]
ω	rotation angle (in radian) of the pile

Superscripts

'+'	indicate a positive value
'–'	indicate a negative value
'FreH'	indicates free-head piles
Bar '–'	denotes normalised values (see Table 1)

bending moment in piles rather than against the applied bending moment.

The 52 p_u profiles deduced from test piles (e^+) in clay, sand or multi-layered soil [13] allow the inadequacy of some prevalent p_u profiles to be revealed. To obtain p_u profile for an anchored pile or its like, pertinent literature for piles, caissons and footings in cohesive soil are reviewed herein. Murff and Hamilton [16] gained an elegant solution for estimating the p_u profile along rigid piles, but for the inability to incorporate the reverse resistance observed above pile-tip level. Aubeny et al. [17] conducted the FEM and PLA analyses on laterally loaded caissons with a slenderness ratio l/d of 2–10. They demonstrated (i) a ~10% variation in the lateral capacity from anisotropic to isotropic strength profiles (smaller variation for $l/d > 6$, and no gapping between caisson and soil); (ii) a normalised rotation depth z_r/l of 0.74–0.78 (FEM), or 0.70–0.76 (PLA) upon loading at mudline (without gapping between caisson and soil); (iii) a normalised capacity $H_o/(s_u dl)$ (i.e. N_p) of 4.2–4.8 (without gapping) or 2.3–3.5 (with gapping), respectively for $l/d = 2$–10 and $e = 0$ (e is a real or fictitious eccentricity of the load above ground level; H_o is the lateral capacity; s_u is undrained shear strength), which are rather close to those gained for anchor plates in clay [18]; and (iv) an $H_o/(s_u dl)$ of 10.2–11.9 (without gapping) or 5.2–8.6 (with gapping), respectively for loading at mid-depth ($e/l = -0.5$). Yun and Bransby conducted FEM analysis on footing in cohesive soil [19], which resembles a short-rigid pile (with a full-length gap on one side). They also provided normalised capacity. These values of normalised capacity reviewed will be used to deduce the values of the factor N_p for caissons, footings and anchored piles in cohesive soil.

The study to date has revealed that under a lateral load H_t and moment loading M_o (=$H_t e$), response of lateral piles [see Fig. 1a] is readily captured using a load transfer approach [14], underpinned by a series of springs (with a subgrade modulus k) distributed along the pile shaft (with limited impact of interaction among the springs). Each spring has a limiting force per unit length p_u [varying with depth z, see Fig. 1b, c_1 and c_2]. Using the ideal elastic–plastic p–$y(u)$ curves, Guo [20,10] developed nonlinear closed-form solutions for free-head, rigid piles. The solutions are associated with mobilisation of the *on-pile force* per unit length (p) along the *limiting* p_u in the depth zones of $0 \sim z_o$ and $z_1 \sim l$, as indicated by the solid lines in Fig. 1c_1 for a Gibson p_u profile, and in Fig. 1c_2 for a constant p_u profile, respectively. The two p_u profiles generally bracket real p_u distributions with depth along piles [21,22]. The associated solutions well capture pile response in comparison with those estimated using empirical p–y curves, despite use of only two input-parameters k and p_u [13]. Two limiting states are defined: (i) *Tip-yield state* at which the on-pile force p at the pile-tip just attains the limiting p_u with $p = p_u$ at $z_1 = l$ [Fig. 1c_1 or c_2]; and (ii) the unachievable *Yield at rotation point (YRP) state* with the slip depths z_o, z_1 merging with the depth of rotation z_r [i.e. $z_o = z_r = z_1$]. The values of lateral loads H_t at the two states are taken as loading capacities H_o.

This paper provides new expressions for estimating bearing capacity of anchored-piles, and develops new solutions for rigid piles with rotating cap for constant or Gibson p_u profile. The expressions are compared with centrifuge tests, numerical (FEM, PLA) results, and complicated SM (rotation-dependent) solutions, and are subsequently used to examine the impact of p_u profiles (with constant k), loading eccentricity, and yielding (at tip and rotation point) states on the capacity, displacement and rotation. The new solutions are used to examine the impact of rotational stiffness of pile-cap on nonlinear response, and to deduce the