Computers and Geotechnics 54 (2013) 117–124

Contents lists available at SciVerse ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Progressive failure analysis of shallow foundations on soils with strain-softening behaviour ☆

E. Conte, A. Donato, A. Troncone *

Department of Civil Engineering, University of Calabria, Rende (Cosenza), Italy

ARTICLE INFO

Article history:
Received 28 January 2013
Received in revised form 7 May 2013
Accepted 4 July 2013
Available online 31 July 2013

Keywords:
Shallow foundations
Bearing capacity
Strain-softening behaviour
Progressive failure
Non-local elasto-viscoplastic model

ABSTRACT

This paper presents a finite element approach to analyse the response of shallow foundations on soils with strain-softening behaviour. In these soils, a progressive failure can occur owing to a reduction of strength with increasing the plastic strains induced by loading. The present approach allows this failure process to be properly simulated by using a non-local elasto-viscoplastic constitutive model in conjunction with a Mohr–Coulomb yield function in which the shear strength parameters are reduced with the accumulated deviatoric plastic strain. Another significant advantage of the method is that it requires few material parameters as input data, with most of these parameters that can be readily obtained from conventional geotechnical tests. To assess the reliability of the proposed approach, some comparisons with experimental results from physical model tests are shown. A fairly good agreement is found between simulated and observed results. Finally, the progressive failure process that occurs in a dense sand layer owing to loading is analysed in details, and the main aspects concerning the associated failure mechanism are highlighted.

1. Introduction

In the current applications, the bearing capacity of shallow foundations is usually evaluated using the well-known equation proposed by Terzaghi [1], i.e.

$$q_{\lim} = qN_q + c'N_c + \frac{1}{2}\gamma BN_\gamma \tag{1}$$

in which $q_{\lim}$ is the bearing capacity pressure, q is the uniformly distributed surcharge replacing the overburden soil at the level of the foundation base, c' is the cohesion intercept of the soil, B is the foundation width, γ is the soil unit weight, N_q, N_c and N_γ are the bearing capacity factors which depend on the soil shearing resistance angle, φ'. Eq. (1) refers to strip footings resting on homogeneous and dry soil with centrally located vertical load and symmetrical failure pattern. To extend Terzaghi's solution to more general conditions than those above specified, a great number of theoretical studies were conducted in which numerical techniques were used to account for reliably the effects of important factors on the bearing capacity calculation, such as footing shape, roughness of base, inclination and eccentricity of loading, ground surface inclination, groundwater conditions and other factors [2–26]. In most of these studies, it was assumed that failure occurs simultaneously along the slip surfaces that develop in the soil mass beneath the footing. However, this failure process is progressive in nature owing to the fact that the plastic strains induced in the soil by loading are markedly non-uniform. As a consequence, the soil shear strength is not simultaneously mobilised at all points of a slip surface. It was also assumed that the soil strength parameters remain unchanged even if large strains are induced by loading. This assumption is inadequate for soils that are characterized by a pronounced strain-softening behaviour, such as dense sands. In reality, in these materials it occurs that some portions of the soil first fail owing to loading, with the shear strain that is located in a zone of limited thickness (shear band). With increasing strain within this zone, soil strength reduces from peak towards the critical state. Owing to the consequent redistribution of stress, the shear band propagates in the soil and a slip surface progressively develops up to causing the collapse of the soil-foundation system. At failure, the average strength mobilised along the slip surface is generally less than the peak strength and greater than the strength at the critical state of the sand under consideration.

The occurrence of a progressive failure in loading tests and centrifugal tests concerning footings on granular soils, was observed by several authors [27–29]. On the basis of these evidences, Perkins and Madson [29] proposed a semi-empirical procedure to estimate the bearing capacity of footings on dense sands. However, a progressive failure process can be successfully predicted using an approach that accounts for properly the strain-softening behaviour of

* Corresponding author. Tel.: +39 0984496533; fax: +39 0984496533.
E-mail address: antonello.troncone@unical.it (A. Troncone).

0266-352X/$ - see front matter © 2013 The Authors. Published by Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compgeo.2013.07.002

the soil and it is able to simulate reliably the formation and development of shear zones within the soil. Finite element approaches with these characteristics were employed by Siddiquee et al. [30] and Banimahd and Woodward [31] to analyse the response of footings on granular soils to loading. Generalised elasto-plastic strain-softening models are incorporated in these approaches for modelling the behaviour of the soil involved. However, these models generally require that a significant number of specific constitutive parameters is determined.

In this study, a finite element approach is proposed to analyse the response of shallow foundations resting on soils with strain-softening behaviour. This approach utilises a non-local elasto-viscoplastic constitutive model in conjunction with a Mohr–Coulomb yield function in which the strength parameters are reduced with the accumulated deviatoric plastic strain. The proposed method requires few material parameters as input data. In addition, most of these parameters can be readily obtained from conventional geotechnical tests. The method is first applied to simulate the experimental results from some physical model tests concerning a rigid strip footing resting on a layer of dense sand in which a thin layer of weak material is located at different depths from the ground surface. Then, the progressive failure process that occurs in a homogeneous sand owing to loading is analysed, and the main aspects of this process are discussed.

2. Non-local elasto-viscoplastic model

Under the assumption of small strains, the total strain rate tensor for an elasto-viscoplastic material can be written as follows:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} \tag{2}$$

where $\dot{\varepsilon}_{ij}$ is the total strain rate tensor, and $\dot{\varepsilon}^{e}_{ij}$ and $\dot{\varepsilon}^{vp}_{ij}$ are the elastic and the viscoplastic strain components, respectively. The tensor $\dot{\varepsilon}^{e}_{ij}$ is defined as

$$\dot{\varepsilon}^{e}_{ij} = C_{ijhk}\dot{\sigma}'_{hk} \tag{3}$$

where $\dot{\sigma}'_{hk}$ is the effective stress rate tensor, and C_{ijhk} is the elastic compliance tensor.

In the present study, the viscous component of the model is used with the primary objective of regularising the numerical solution, as it was suggested by Zienkiewicz and Cormeau [32]. Following Perzyna [33], the viscoplastic strain rate tensor is expressed by the following equation:

$$\dot{\varepsilon}^{vp}_{ij} = \Phi(F)m_{ij} \tag{4}$$

in which Φ is the viscous nucleus that depends on the yield function F, and m_{ij} is the gradient to the plastic potential function Q (i.e., $m_{ij} = \partial Q/\partial\sigma'_{ij}$). The gradient of Q defines the direction of the viscoplastic strain rate tensor, and the yield function influences the modulus of this tensor by means of Φ. In this connection, di Prisco and Imposimato [34] proposed the following relationship for Φ:

$$\Phi(F, p') = \bar{\gamma}p'e^{\bar{\alpha}F} \tag{5}$$

where $\bar{\gamma}$ and $\bar{\alpha}$ are constitutive parameters, and p' denotes the mean effective stress. A maximum value of 3 should be assumed for the product $\bar{\alpha}F$ to prevent the exponent in Eq. (5) from becoming excessively large [36]. The parameter $\bar{\gamma}$ influences the strain rate and consequently the rapidity with which strain occurs owing to a given stress increment. In particular, strain rate increases with increasing the value of $\bar{\gamma}$ [36]. The Mohr–Coulomb failure criterion is adopted in the present study to describe the yield function F. Referring to cohesionless soils, the expression of F in terms of the principal effective stresses is

$$F = \frac{1}{2}(\sigma'_1 - \sigma'_3) - \frac{1}{2}(\sigma'_1 + \sigma'_3)\sin\varphi' \tag{6}$$

where σ'_1 and σ'_3 are the major and minor principal effective stresses respectively (it is assumed that compressive stress is positive). Following several authors [18,37–41], the strain-softening behaviour of the soil is simulated by reducing the soil shearing resistance angle φ' from the peak value, $\varphi'_{\rm p}$, to that at constant volume, $\varphi'_{\rm cv}$, with increasing the accumulated deviatoric plastic strain. In the present study, this strain is expressed by the parameter $k_{\rm shear}$ that is defined as follows [42]:

$$k_{\rm shear} = \int \dot{k}_{\rm shear}dt \tag{7}$$

where

$$\dot{k}_{\rm shear} = \sqrt{0.5\dot{e}^{\rm p}_{ij}\dot{e}^{\rm p}_{ij}} \tag{8}$$

t is time, and $\dot{e}^{\rm p}_{ij}$ is the deviatoric plastic strain rate tensor the expression of which is

$$\dot{e}^{\rm p}_{ij} = \dot{\varepsilon}^{\rm p}_{ij} - \frac{1}{3}\dot{\varepsilon}^{\rm p}_{kk}\delta_{ij} \tag{9}$$

in which δ_{ij} is the Kronecker tensor, and $\dot{\varepsilon}^{p}_{ij}$ is the plastic deformation rate tensor. For simplicity, the relationship considered in the present study to relate the mobilised shearing resistance angle to $k_{\rm shear}$ is schematised in Fig. 1a. As can be seen, this relationship is defined by two thresholds denoted as $k^{\rm p}_{\rm shear}$ and $k^{\rm cv}_{\rm shear}$, respectively.

The flow rule is of non-associated type with the plastic potential function which is expressed as

$$Q = \frac{1}{2}(\sigma'_1 - \sigma'_3) - \frac{1}{2}(\sigma'_1 + \sigma'_3)\sin\psi \tag{10}$$

in which ψ denotes the dilatancy angle of the soil. A relationship similar to that shown in Fig. 1a is also considered for ψ (Fig. 1b).

A significant advantage of this constitutive model is that it requires few soil parameters as input data, in comparison with other more sophisticated models. Specifically, these parameters are Young's modulus E', Poisson's ratio ν', the parameters defining the shear strength of the soil (i.e., $\varphi'_{\rm p}$ and $\varphi'_{\rm cv}$), the dilatancy angle $\psi_{\rm P}$ at peak, the strain thresholds, $k^{\rm p}_{\rm shear}$ and $k^{\rm cv}_{\rm shear}$, and the viscous parameters $\bar{\gamma}$ and $\bar{\alpha}$. Generally, the most part of these parameters are directly obtained from triaxial tests. In addition, considering the role principally attributed to the viscous component of the constitutive model (i.e., to regularise the numerical solution [32]), the

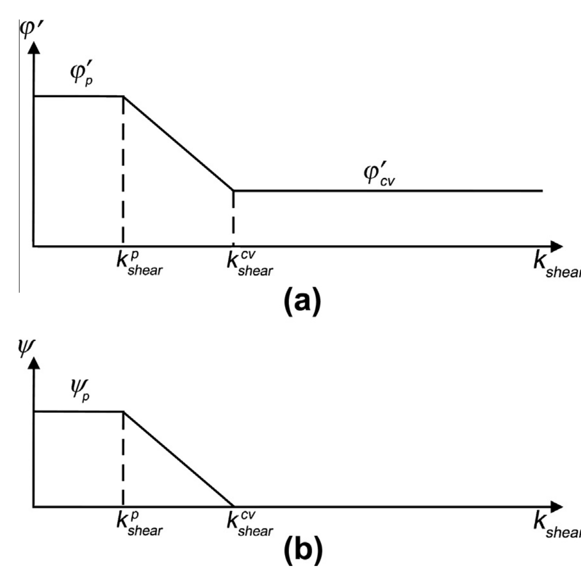

Fig. 1. (a) Adopted φ'-k_{shear} relationship with an indication of the strain thresholds, k^{p}_{shear} and k^{cv}_{shear}, and (b) adopted ψ-k_{shear} relationship.