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Particle finite element analysis of large deformation and granular flow problems

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ABSTRACT

A version of the Particle Finite Element Method applicable to geomechanics applications is presented. A simple rigid-plastic material model is adopted and the governing equations are cast in terms of a variational principle which facilitates a straightforward solution via mathematical programming techniques. In addition, frictional contact between rigid and deformable solids is accounted for using an approach previously developed for discrete element simulations. The capabilities of the scheme is demonstrated on a range of quasi-static and dynamic problems involving very large deformations.

1. Introduction

Although most geotechnical structures operate in the small deformation range, there are numerous problems within geotechnical engineering and related fields that call for methods where changes to the problem geometry as a result of deformations are taken into account. These include problems of landslides and debris flow, penetration of various devices such as cones and torpedo anchors into the ground, and the interaction, during installation or under operating conditions, of off-shore oil and gas infrastructure with the seabed.

For some of these problems, the deformation pattern resembles a fluid flow more than a solid undergoing large distortions. In the framework on the standard finite element method, such problems give rise to two fundamental challenges. The first one relates to geometry in the sense that the magnitude of the deformations is bound to lead not only to severe mesh distortion, but also to situations where the boundaries of the problem change from one time step to the next. Of these two separate but related issues, the former has received by far the most attention. Indeed, for many problems the original boundaries are maintained even after relatively large distortions. The perhaps most common approach to avoiding or alleviating mesh distortion is the Arbitrary Eulerian Lagrangian (ALE) method. This method utilizes the respective advantages of pure Eulerian and pure Lagrangian formulations and has been used quite successfully in geotechnical applications [1,2] and other solid as well as fluid mechanics problems [3,4]. Another popular method for geotechnical applications is the so-called Remeshing and Interpolation Technique with Small Strain (RITSS) technique proposed by Hu and Randolph [5,6]. While both the ALE and the RITSS have been used to solve problems involving relatively large deformations, they both have shortcomings in the case where the original boundaries change in the course of the deformation process, for example in the case where an initially contiguous solid separates into two or more parts as a result of external actions.

The second challenge, which in many ways is the more serious one (though it remains much less explored), is that of solving the governing equations – comprising momentum balance, strain–displacement relations and constitutive relations that usually are highly nonlinear and may give rise to ill-posedness, localization of deformations, etc. Indeed, it is well known that even small deformation problems are hard to deal with for constitutive models that involve nonassociated flow rules [7].

In this paper a new scheme that addresses both of the fundamental challenges described above is presented. The scheme is applicable to general large deformation problems with no real limitations on the magnitude of the deformations. In other words, both problems where the deformation patterns resemble fluid flows and those that merely involve the deformation of solids slightly beyond the small deformation limit can be handled. More specifically, issues related to geometry are handled by means of the Particle Finite Element Method (PFEM) [8–10] while the solution of the governing equations is addressed by means of variational and

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mathematical programming methods that have their origin in computational limit analysis [11–16] but since have been applied to a wide range of other problems including elastoplasticity [17,18,7] and discrete element type analysis [19–21]. Other issues dealt with include dynamics and frictional contact.

The paper is organized as follows. In Section 2, the fundamentals of the PFEM are briefly described with emphasis on the alpha-shape method used for identifying solid and void domains on the basis of a cloud of points. Section 3 details the governing equations and their variational formulation. Next, in Section 4, the discretization and solution of the governing equations are described before the treatment of contact is detailed in Section 5. Finally, in Section 6, a number of examples demonstrating the capabilities of the scheme are presented before conclusions are drawn in Section 7. While all aspects of the new scheme are applicable to the general three-dimensional setting, the examples given in this paper are limited to two dimensions assuming plane strain.

Standard matrix notation is used throughout with bold upper and lower case letters denoting matrices and vectors respectively.

2. Particle Finite Element Method

The Particle Finite Element Method (PFEM) is, despite its name, a mesh based continuum method. First developed for fluid dynamics applications [8–10], the PFEM makes use of a Lagrangian description to account for the motion of nodes of the finite element mesh. A key feature of the method is that nodes are viewed as free 'particles' that can separate from the solid to which they originally belong. On the basis of the resulting cloud of points, the solid and void domains are identified and a standard finite element discretization used to advance the simulation by a given time step. More specifically, considering a time step $t_n \rightarrow t_{n+1}$, the steps of the PFEM are as follows (see also Fig. 1):

0. A cloud of particles, C^n, is given at time t_n.
1a. On the basis of C^n, identify the computational domain, V^n.
1b. Mesh the domain and discretize the governing equations on M^n.
1c. Map the state variables (velocities, stresses, etc.) from the old mesh, M^{n-1}, to the new mesh, M^n.
2a. Solve the discrete governing equations to obtain the displacement of the nodes.
2b. Update the positions of the nodes to arrive at C^{n+1} and repeat.

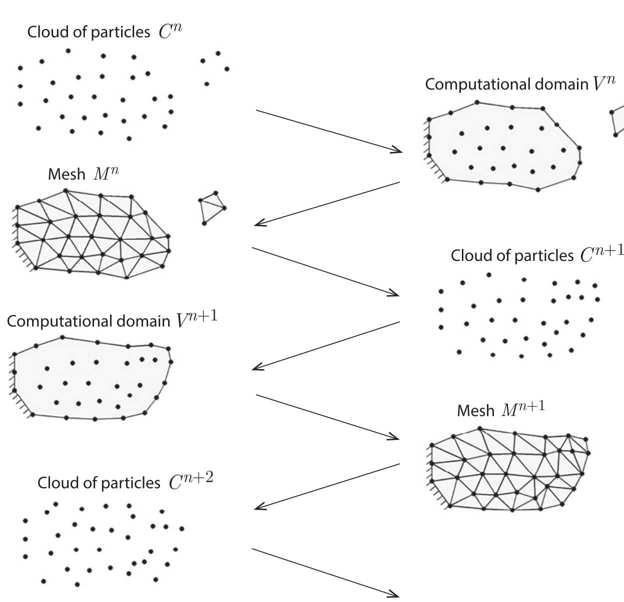

Fig. 1. Steps in the Particle Finite Element Method.

2.1. Alpha-shape method

The critical issue in the steps above is the identification of the computational domain on the basis of a cloud of points. For the general case, there is no unique solution to this problem. The solution originally proposed by Idelsohn et al. [8] and subsequently adopted as a standard feature of the PFEM was to use the alpha-shape method previously developed by Edelsbrunner and Mucke [22] for computer graphics applications.

The basic principle of the alpha-shape method is as follows. Consider a cloud of points with a characteristic spacing h. Then for some predefined value of a parameter α, *all nodes on an empty sphere with a radius greater than* αh *are considered boundary nodes*. In other words: for each point in the domain examine whether it is possible to place a sphere with radius αh such that it contains only that point. If possible, the point is a boundary point and if not, i.e. if the sphere inevitably will contain more than the one point, it is an internal point.

While a number of algorithms for recognizing boundaries by means of the alpha-shape method are available, another, and in many ways more straightforward possibility, has been proposed by Cremonesi et al. [23]. The steps in this scheme are as follows. Consider the cloud of particles as shown in Fig. 2(a). A Delaunay triangulation is first performed to generate the convex domain shown in Fig. 2(b). Next, the radius of the circumcircle of each triangle is examined and triangles with a circumcircle radius greater than αh are deleted. For the example at hand, this leads to the final configuration shown in Fig. 2(c). As shown by Cremonesi et al., this procedure is equivalent to the original alpha-shape approach owing to the property that the circumcircles of all triangles generated by the Delaunay triangulation are empty.

2.1.1. Choice of α

It is clear that the alpha-shape method involves an element of subjectivity. Indeed, the resulting domain is a direct function of the value of the parameter α. This is illustrated in Fig. 3. A value of $\alpha = 1.2$ here produces a set of boundaries that in many cases would be deemed reasonable. In fact, any value of α in the interval $0.9 \leqslant \alpha \leqslant 1.3$ produces this set of boundaries. Decreasing α below 0.9 leads first to internal voids ($\alpha = 0.5$) and subsequently to a disintegration of the external boundaries as well ($\alpha = 0.4$). On the other hand, increasing α above 1.3 leads first to a coalescence of the two distinct sets of points ($\alpha = 1.5$) and then, for large values of α, to a solid defined by the convex hull inscribing the cloud of points.

The conclusion of this example, that a value of α slightly greater than 1 is appropriate, is consistent with experience from application to actual physical problems. For example, for a wide range of coupled fluid–solid interaction problems, Onate et al. [24] conclude that a value of α in the range of 1.3–1.5 is appropriate. For

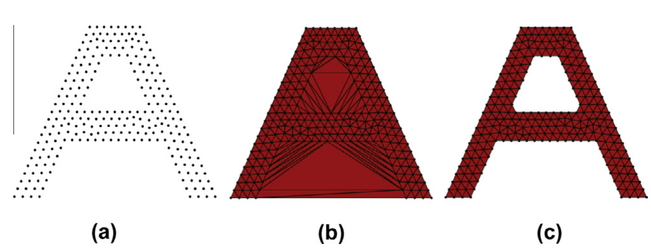

Fig. 2. Boundary recognition via the scheme of Cremonesi et al. [23]: cloud of points (a), Delaunay triangulation (b), after deletion of triangles with circumcircle greater than αh (c).