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# Finite-difference model for one-dimensional electro-osmotic consolidation

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ABSTRACT

Small strain consolidation theories treat soil properties as being constant and uniform in the course of consolidation, which is not true in the case of electro-osmosis-induced consolidation practices. Electro-osmotic consolidation leads to large strain, which physically and electro-chemically affects to a non-negligible extent the nonlinear changes of the soil properties. For the nonlinear changes, iterative computations provide a mathematical approximation of the soil consolidation when the time steps and spatial geometry are intensively meshed. In this context, this paper presents a finite-difference model, EC1, for one-dimensional electro-osmotic consolidation, and this model is developed based on a fixed Eulerian co-ordinate system and uses a piecewise linear approximation. The model is able to account for the large-strain-induced nonlinear changes of the physical and electro-chemical properties in a compressible mass subjected to electro-osmotic consolidation and to predict the consolidation characteristics of the compressible mass. EC1 is verified against exact analytical solutions and test results obtained from an experimental program. Example problems are illustrated with respect to the numerical solutions of large-strain electro-osmotic consolidation.

## 1. Introduction

Electro-osmosis is a process enabling the flow of pore fluid in a soil mass in the direction toward a negative electrode (cathode) in response to a voltage gradient (electrical field or potential difference) of direct current that is applied between the cathode and a positive electrode (anode). The cathode and anode are installed in pairs in the soil mass, between which electrical current is transmitted primarily by the movement of ions through the pore fluid. The capacity of electro-osmosis is employed in many geotechnical and geoenvironmental practices, such as soil remediation, site reclamation and ground dewatering, where clays or other very low permeability materials are intensively deposited and the uses of conventional soil treatment technology are less efficient. Though the liquid and solid phases of the soil mass are taken to be incompressible, the consolidation induced by electro-osmosis may subject the soil skeleton to significant compression for high moisture content fine-grained soils, e.g., newly reclaimed or dredged coastal sediments, municipal sludge and industry solid–liquid mixed disposals. That is, the soil properties undergo changes under electro-osmotic consolidation, which has been noted in many previous experiments [1–4]. The changes may be significant and non-negligible, so small strain electro-osmotic consolidation theories [5–7], which usually assume that the soil's physical and electro-chemical properties are uniform throughout the soil matrix and constant over time, are not as applicable. The changes in the soil properties are predictable within engineering accuracy once the factors that cause the changes are understood. The changes can be integrated into a computational program to refine the approximation of consolidation. In contrast to small strain electro-osmotic consolidation theories where soil properties are assumed unchanged, large strain is taken into consideration for electro-osmotic consolidation to account for changes in soil properties.

For the approximation of large strain consolidation, a suitable point of departure involves a piecewise-linear approximation that is based on Eulerian co-ordinates. Compared with theories [8–10] based on Lagrangian co-ordinates developed to approximate large strain consolidation, the advantages of developing Eulerian co-ordinate-based models include greater versatility regarding initial conditions, boundary conditions and soil heterogeneity [11]. In the piecewise-linear approach, finite elements are integrated over the material's co-ordinate space, whereas in finite differencing, the elements are integrated over time. After each time step, all variables pertaining to the problem geometry, material properties, fluid flow and effective stress are updated with respect to a fixed Eulerian co-ordinate system [12,13]. The time increment of each step must be sufficiently small so that all variables can be approximated as constants for each iteration. This constraint is a limitation of the piecewise-linear method. However, past studies [11,14–16] have

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## Nomenclature

| | |
|---|---|
| $a$ | experimental derived exponent of the coefficient of electro-osmotic permeability |
| $a_v$ | coefficient of compressibility |
| $A_j$ | average area of the cross section between contiguous elements, $j$ and $(j-1)$ |
| $C_c$ | compression index |
| $C_k$ | hydraulic permeability index |
| $D$ | dielectric constant of pore fluid |
| $e$ | void ratio |
| $e^*$ | threshold void ratio |
| $E$ | energy consumption index |
| $G_s$ | specific gravity of soil solids |
| $h$ | total head of water |
| $h_{w1}$ | total head of water adjacent to cathode |
| $h_{w2}$ | total head of water adjacent to anode |
| $H_0$ | initial height of compressible soil mass |
| $i_e$ | voltage gradient |
| $i_h$ | hydraulic gradient |
| $j$ | element co-ordinate |
| $k$ | coefficient of hydraulic or electro-osmotic permeability |
| $k_e$ | coefficient of electro-osmotic permeability |
| $k_{es}$ | equivalent series coefficient of electro-osmotic permeability |
| $k_h$ | coefficient of hydraulic permeability |
| $k_{hs}$ | equivalent series coefficient of hydraulic permeability |
| $k^*$ | threshold coefficient of permeability |
| $l_0$ | thickness of element |
| $L$ | length of compressible soil mass |
| $m$ | loop calculation termination variable |
| $n$ | soil porosity |
| $p$ | stress/load in oedometer test |
| $q$ | volume of flow |
| $q_h$ | flow volume induced by hydraulic gradient |
| $q_e$ | flow volume induced by electrical gradient |
| $R_j$ | number of elements for soil mass |
| $R_m$ | number of piece-linear points in compressibility constitutive relationship curves |
| $R_n$ | number of piece-linear points in permeability constitutive relationship curves |
| $s_{avg}$ | average settlement of soil mass |
| $s_j$ | settlement of element $j$ |
| $t$ | elapsed time of consolidation |
| $t_f$ | final elapsed time |
| $T_v$ | time factor |
| $u$ | pore pressure |
| $U_{avg}$ | average consolidation degree |
| $V_j$ | electrical potential at element $j$ |
| $V_m$ | electrical potential difference between electrodes |
| $V_v$ | volume of soil mass |
| $w$ | water content |
| $w_p$ | plastic limit |
| $w_L$ | liquid limit |
| $W$ | width of compressible soil mass |
| $x$ | distance to cathode |
| $z_{c,j}$ | elevation of upper corner of element $j$ |
| $z_j$ | elevation of node of element $j$ |
| $\sigma$ | total stress |
| $\sigma'$ | effective stress |
| $\eta$ | viscosity of pore fluid |
| $\gamma$ | unit weight of soil |
| $\gamma_w$ | unit weight of water |
| $\xi$ | soil zeta potential |
| $\rho$ | electrical resistivity |
| $\rho_s$ | electrical resistivity of soil solid particles |
| $\rho_w$ | electrical resistivity of pore water |

shown that the piecewise-linear method compares favorably to other large strain formulations and is able to yield validated results with a numerical simulation.

This paper presents a piecewise linear numerical model, called Electro-osmotic Consolidation 1 (EC1), to describe one-dimensional electro-osmotic consolidation. This model is developed with the aid of model CS2 [15]. CS2 is a model to approximate vertical consolidation settlement of compressible soil layer. Similar to CS2, EC1 is able to account for a large strain, the soil self-weight, the relative velocity of the fluid and solid phases, and the nonlinear variation of the soil properties (compressibility, hydraulic and electrical conductivity) associated with electro-osmotic consolidation. The constitutive relationships for the soil properties are specified using discrete data points extracted from mathematical approximations or derivative functions of soil properties. The performance of EC1 is verified by comparing its numerical solutions to exact analytical solutions and experimental test results. Example problems involving large-strain settlement and the nonlinear constitutive relationships are illustrated to show the progress of electro-osmotic consolidation. The study presented in this paper continues and complements the content published in one of the authors' recent papers [17], particularly with respect to the experimental setups, the validation of the model and the numerical output results for the example problems.

## 2. Model description

### 2.1. Geometry

The initial geometry of a compressible mass prior to the application of a voltage gradient (time $t < 0$) is shown in Fig. 1a.

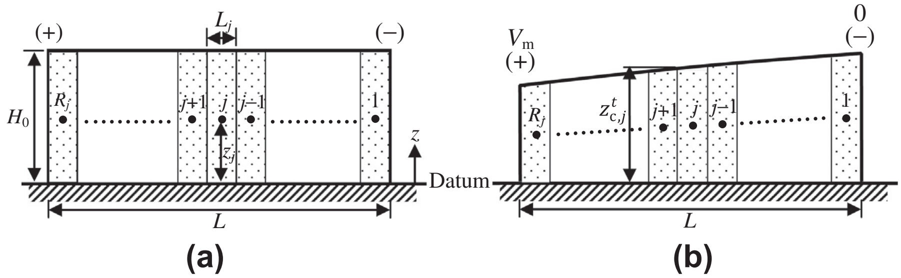

**Fig. 1.** Geometry for EC1: (a) initial configuration ($t < 0$) and (b) configuration after the application of the voltage gradient ($t \geqslant 0$) [17].