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One-dimensional consolidation of layered soils with exponentially time-growing drainage boundaries

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ABSTRACT

Fully drained and undrained boundary conditions are commonly applied to solve the consolidation problems. In most practical situations, however, impeded drainage boundaries are really a matter of great concern. As an attempt to idealize such boundary conditions, a time-decaying exponential function has recently been suggested to describe the changes of excess pore water pressure at the boundaries of consolidating soils subjected to instantaneous loading. It allows the excess pore water pressure to dissipate smoothly rather than abruptly from its initial value given by the instantaneous loading to the value of zero, leading to an exponentially time-growing drainage boundary. In this study, a numerical solution of one-dimensional consolidation of layered soils with such defined boundaries is derived by using the method of Laplace transform and its numerical inverse. The solution is explicitly expressed and conveniently coded into a computer program for ease and efficiency of practical use. Analytical solutions of one-dimensional consolidation of single-layered soil are also derived by using the method of analytical inverse Laplace transform and the method of separation of variables as well. By comparing these two analytical solutions and comparing with an available analytical solution for two-layered soil with pervious boundaries, the proposed numerical solution for layered soils is validated. Good agreement is obtained, and the accuracy of the numerical solution is verified. Moreover, the dissipation of excess pore water pressure and the increase of degree of consolidation with time for a four-layered soil are investigated to assess the effects of the adopted drainage boundary conditions on consolidation.

1. Introduction

Natural soils are often composed of several soil layers with different characters. Considering the varieties of soil stratigraphy, it may not be possible to simplify the compressible stratum in the field as a single one soil layer [1,2]. Several analytical solutions for one-dimensional consolidation of layered soils have been developed [1,3–7]. The drainage boundaries are normally treated as pervious or impervious, as expressed as follows:

$$\text{Pervious}: \quad u = 0 \tag{1}$$

$$\text{Impervious}: \quad \frac{\partial u}{\partial z} = 0 \tag{2}$$

where u is the excess pore water pressure at the boundary of consolidating soils, and z is the vertical coordinate.

In most practical consolidation problems, however, the drainage at the boundaries of consolidating soils is impeded [8]. Gray [8] suggested that the boundary conditions of impeded drainage can be described as follows:

$$\frac{\partial u}{\partial z} = R\frac{u}{H} \tag{3}$$

where H is the thickness of consolidating soils and R is a dimensionless parameter reflecting the relative permeability of the boundary to the consolidating soils. When R tends towards infinity, Eq. (3) approaches the pervious boundary condition, i.e., Eq. (1). When R equals zero, Eq. (3) represents the impervious boundary condition, i.e., Eq. (2). Based on Eq. (3), solutions of one-dimensional consolidation have been derived for one- [8,9], two- [10] and multi-layered soils [1,11,12]. Nevertheless, Lee et al. [3] pointed out that the impeded drainage boundary conditions can be taken as special cases of permeable, yet incompressible, top or bottom soil layer in a multi-layered system.

Recently, Mei et al. [13] have put forward an exponential function to describe the impeded drainage boundary condition, as

$$u = q_0 e^{-at} \tag{4}$$

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where q_0 is the initial excess pore water pressure at the boundary of consolidating soils; e is the base of the natural logarithm; t is the dissipation time and a is the parameter reflecting the dissipation rate of excess pore water pressure at the boundary. When a tends towards infinity, Eq. (4) approaches the pervious boundary condition, i.e., Eq. (1). For a given value of a, the excess pore water pressure at the boundary will dissipate with time from an initial value of q_0 to the value of zero. By using Eq. (4), an exponentially time-growing drainage boundary condition is assumed. For practical applications of Eq. (4), the parameter a can be determined by curve-fitting the field measurements at the boundaries of consolidating soils. Based on Eq. (4), Mei et al. [13] derived an analytical solution of one-dimensional consolidation of single-layered soil.

The objective of this paper is to investigate the one-dimensional consolidation behavior of layered soils under the assumed boundary conditions of Eq. (4). By using the method of Laplace transform with respect to time, an explicit analytical solution to the governing equation is derived. Numerical inversion of the Laplace transform in the time domain is then applied to obtain the solution of excess pore water pressure. The validity and the accuracy of the numerical solution are verified against two analytical solutions derived for single-layered soil with the same drainage boundaries, and against an available analytical solution for two-layered soil with pervious top and bottom boundaries. In addition, the influence of the adopted impeded drainage boundary conditions on the consolidation behavior of a four-layered soil is investigated.

2. Problem description

The system consisting of n contiguous homogeneous soil layers with impeded drainage boundaries is shown schematically in Fig. 1. The soil layer is indexed with i (=1,2,...,n). For the ith layer of soil, h_i, k_{vi} and E_{si} represent, respectively, the thickness, the hydraulic conductivity and the modulus of compressibility of the soil. A uniform instantaneous loading q_0 is assumed applied on the top of the top soil layer. All the assumptions made in Terzaghi's consolidation theory are retained except that Eq. (4) is assumed as the drainage boundary conditions. The governing equation of one-dimensional consolidation of the ith soil layer is given by

$$\frac{\partial u_i}{\partial t} = c_{vi}\frac{\partial^2 u_i}{\partial z^2} \quad i = 1, 2, \ldots, n \tag{5}$$

where u_i is the excess pore water pressure of the ith soil layer; c_{vi} = -$k_{vi}E_{si}/\gamma_w$ is the coefficient of consolidation of the ith soil layer; γ_w is the unit weight of water; and z is the downward vertical coordinate originated from the top boundary.

The continuity of excess pore water pressure and the continuity of flow rate at the interfaces between adjoining soil layers can be expressed as

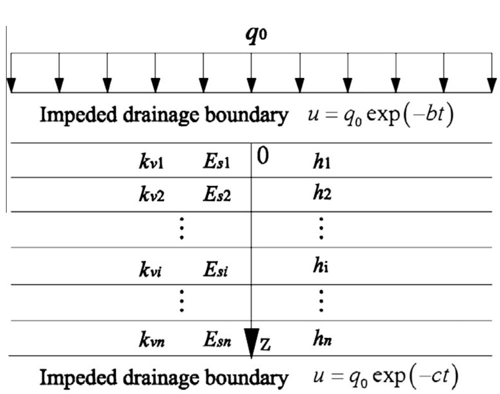

Fig. 1. Profile of layered soils with impeded drainage boundaries.

$$u_i|_{z=H_i} = u_{i+1}|_{z=H_i} \quad i = 1, 2, \ldots, n-1 \tag{6}$$

$$k_{vi}\frac{\partial u_i}{\partial z}\bigg|_{z=H_i} = k_{v(i+1)}\frac{\partial u_{i+1}}{\partial z}\bigg|_{z=H_i} \quad i = 1, 2, \ldots, n-1 \tag{7}$$

where $H_i = \sum_{k=1}^{i} h_k$.

The initial conditions at t = 0, assumed in this case to be uniform with depth, are given by

$$u_i(z, 0) = q_0 \quad i = 1, 2, \ldots, n \tag{8}$$

For generality, the top and bottom drainage boundary conditions are assumed as

$$u_1|_{z=0} = q_0 e^{-bt} \tag{9}$$

$$u_n|_{z=H_n} = q_0 e^{-ct} \tag{10}$$

where b and c are parameters of impeded drainage at the top and bottom boundaries, respectively.

3. Laplace transforms

3.1. Governing equation

Letting $u_i^*(z,t) = u_i(z,t) - u_i(z,0)$ and considering Eq. (8), the following equation can be obtained.

$$u_i(z,t) = u_i^*(z,t) + q_0 \tag{11}$$

Substituting Eq. (11) into Eq. (5) yields

$$\frac{\partial u_i^*}{\partial t} = c_{vi}\frac{\partial^2 u_i^*}{\partial z^2} \quad i = 1, 2, \ldots, n \tag{12}$$

The Laplace transform of $u_i^*(z,t)$ with respect to time t can be written as

$$\tilde{u}_i^*(s) = \int_0^\infty u_i^* e^{-st} \mathrm{d}t \tag{13}$$

where s is a complex number representing the frequency domain or Laplace-space variable.

The Laplace transform of the term on the left-hand side of Eq. (12) with respect to time t is given by

$$\begin{aligned} \int_0^\infty \frac{\partial u_i^*}{\partial t} e^{-st}\mathrm{d}t &= \int_0^\infty e^{-st}\mathrm{d}u_i^* = [e^{-st}u_i^*]_{t=0}^{t=\infty} - \int_0^\infty u_i^* \mathrm{d}e^{-st} \\ &= -u_i^*(z, t=0) + s\int_0^\infty u_i^* e^{-st}\mathrm{d}t = 0 + s\,\tilde{u}_i^* \\ &= s\,\tilde{u}_i^* \end{aligned} \tag{14}$$

And the Laplace transform of the term on the right-hand side of Eq. (12) with respect to time t is given by

$$\int_0^\infty c_{vi}\frac{\partial^2 u_i^*}{\partial z^2} e^{-st}\mathrm{d}t = c_{vi}\frac{\partial^2}{\partial z^2}\int_0^\infty u_i^* e^{-st}\mathrm{d}t = c_{vi}\frac{\partial^2 \tilde{u}_i^*}{\partial z^2} \tag{15}$$

Thus, the Laplace transform of Eq. (12), namely the governing equation of one-dimensional consolidation of layered soils, can be rewritten as

$$\frac{\partial^2 \tilde{u}_i^*}{\partial z^2} = \frac{s}{c_{vi}}\tilde{u}_i^* \tag{16}$$

3.2. Continuity conditions at the soil interfaces

The Laplace transform of Eq. (6) with respect to time t is given by

$$\tilde{u}_i^*|_{z=H_i} = \tilde{u}_{i+1}^*|_{z=H_i} \quad i = 1, 2, \ldots, n-1 \tag{17}$$