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An important pitfall of pseudo-static finite element analysis

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ABSTRACT

Finite Element (FE) pseudo-static analysis can provide a good compromise between simplified methods of dynamic analysis and time domain analysis. The pseudo-static FE approach can accurately model the in situ, stresses prior to seismic loading (when it follows a static analysis simulating the construction sequence) is relatively simple and not as computationally expensive as the time domain approach. However this method should be used with caution as the results can be sensitive to the choice of the mesh dimensions. In this paper two simple examples of pseudo-static finite element analysis are examined parametrically, a homogeneous slope and a cantilever retaining wall, exploring the sensitivity of the pseudo-static analysis results on the adopted mesh size. The mesh dependence was found to be more pronounced for problems with high critical seismic coefficients values (e.g. gentle slopes or small walls), as in these cases a generalised layer failure mechanism is developed simultaneously with the slope or wall mechanism. In general the mesh width was found not to affect notably the predicted value of critical seismic coefficient but to have a major impact on the predicted movements.

1. Introduction

The seismic design of geotechnical structures often relies on simplified pseudo-static methods of analysis. For example, limit equilibrium (LE) based methods, like the Mononobe–Okabe method, are still widely used in engineering practice for the seismic design of retaining structures. These simple design procedures are straightforward, but they do not provide directly any indication of deformations under the design earthquake load. However during an earthquake, movements of both the soil and the structure will occur under seismic loading, regardless of how over-designed the structure may be. To get an estimate of the seismically induced movements the LE methods are usually combined with a sliding block type of analyses which have been shown to be very sensitive to the seismic coefficient obtained by the LE analysis [2].

On the other hand, time domain analysis, using acceleration time histories, provides a rigorous tool for the safe and economic seismic design of a geotechnical structure, as it can give predictions of the performance of a structure under any given seismic scenario. However, this type of analysis requires the use of computational codes (Finite Element (FE)/Finite Difference) which encompass advanced constitutive models capable of simulating the response of soils to seismic loading and boundary conditions specifically formulated for dynamic analysis. Such advanced tools are not usually readily available in engineering practice and the calibration and analysis of the computational models can be time consuming. The use of finite element pseudo-static analysis can be a good compromise between simplified methods of analysis and time domain analysis and consequently is widely used in engineering practice. The pseudo-static FE approach can accurately model the in-situ stresses prior to seismic loading (when it follows a static analysis simulating the construction sequence) is relatively simple and not as computationally expensive as the time domain approach.

Despite its simplicity and the plethora of relevant studies (e.g. [10,6,9,7]), there are still a number of issues related to the pseudo-static finite element approach, and in particular the dependence of the solution on the mesh size, which have not been addressed adequately in the literature. This paper attempts to clarify some of the limitations of pseudo-static FE analysis using two simple examples; a homogeneous slope and an excavation next to a cantilever wall, but most of the issues raised are relevant to other applications of the pseudo-static methodology.

1.1. Pseudo-static finite element analysis

Pseudo-static finite element analysis is used to evaluate the seismic response of various types of geotechnical structures such as retaining walls, embankments, dams, tunnels. Depending on the type and geometry of the problem two approaches of pseudo-static analysis can be followed:

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- *Force based analysis*: In this case, the seismically induced inertia forces are approximated as a constant body force (in one or two directions) which is applied incrementally throughout the whole mesh (see Fig. 1a):

$$F_h = k_h W \tag{1a}$$

$$F_v = k_v W \tag{1b}$$

where F_h, F_v are the horizontal and vertical body forces respectively, k_h, k_v are the corresponding seismic coefficients and W is the weight of the failure mass. The main objective of the analysis is either to determine the critical acceleration ($k_c g$) for which the structure fails or to determine the factor of safety for the design acceleration level.

- *Deformation based analysis*: In this case, the mesh is subjected to simple shear conditions, as schematically illustrated in Fig. 1b. A uniform displacement, u_s, and a triangular displacement distribution, are applied incrementally along the top and the lateral boundaries of the mesh respectively. For the calculation of the displacement u_s a site response analysis is performed first which determines the maximum free-field shear strain (γ_{ff}) at the level corresponding to the structure of interest (see Fig. 1b).

The deformation based analysis simulates more realistically the seismic loading as the imposed deformation is based on a site response analysis which takes into account the dynamic response of the stratigraphy to a time-varying ground motion. However this approach can only be used for problems in which it is possible to impose simple shear conditions to the mesh (mainly seismic analysis of underground structures, e.g. [3,5,1]) and it cannot be used to calculate the factor of safety or the critical failure mode. Therefore, for most problems, the force based approach is followed. The aim of the present study is to highlight some common pitfalls related to the use of the forced based approach by analysing two simple problems investigating the dependence of the solution on the mesh width.

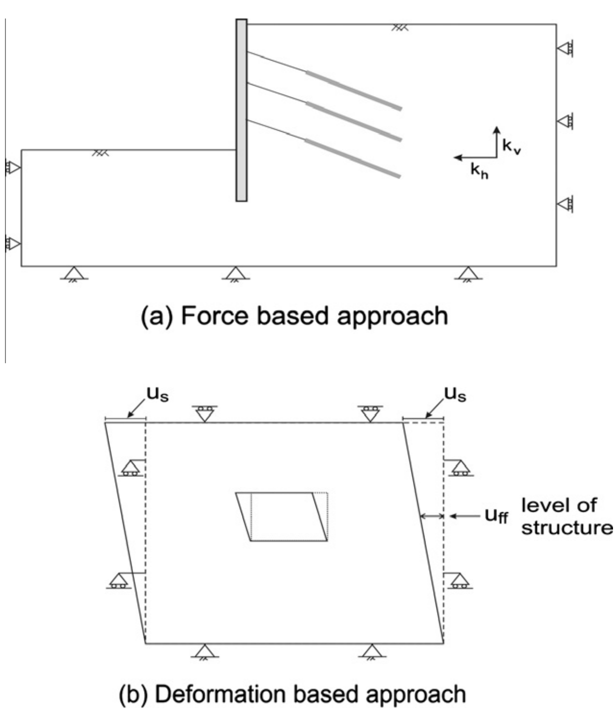

Fig. 1. Schematic representation of FE mesh configuration in pseudo-static analysis; (a) force based approach and (b) deformation based approach, where u_{ff} is the maximum free-field displacement at the level of the structure and u_s is the displacement applied at the top boundary of the mesh.

1.2. Homogeneous layer failure mechanism

Before examining the slope and the retaining wall examples, it is important to establish the failure mechanism which is imposed by the force based approach in a green-field profile. Therefore the first problem analysed consists of a simple homogeneous soil layer of thickness DH overlaying perfectly rigid bedrock, which is subjected to an incremental horizontal body force. The objective of the exercise is to determine the critical horizontal yield acceleration coefficient, k_h^{lim}, for which the layer fails by sliding along the interface with the rigid bedrock. It will be shown in the following examples that this layer mechanism can be practically mobilised in any type of force based pseudo-static analysis and determines the limiting value of pseudo-static horizontal acceleration that can be reached. As suggested by Loukidis et al. [7], considering the limit equilibrium of a homogeneous soil profile of thickness DH overlying a rigid layer, the critical seismic coefficient, k_h^{lim}, required to balance the shear resistance at the interface is given by:

$$k_h^{lim} = \frac{c}{\gamma \mathrm{DH}} + \tan\phi' \tag{2}$$

where γ is the bulk unit weight of the soil, DH is the layer thickness and c, ϕ' are the cohesion and the angle of shearing resistance respectively.

1.2.1. Analysis arrangement

The problem geometry and the adopted boundary conditions, restriction of movement in both directions along the bottom boundary and restriction of the horizontal displacement along the lateral boundaries, are schematically illustrated in Fig. 2. In all the following analyses an initial stress field was assumed adopting the K_o expression of Jaky [4]:

$$K_o = 1 - \sin\phi' \tag{3}$$

The finite element mesh was constructed with 8 noded isoparametric quadrilateral elements and all the analyses presented herein were performed in plane strain with the finite element code ICFEP [8]. The soil was assumed to be dry and was modelled using an associated Mohr-Coulomb failure criterion, while the behaviour is assumed to be isotropic linear elastic before failure. The adopted soil properties are detailed in Table 1. The pseudo-static failure mechanism of the homogeneous soil layer was investigated by subjecting a layer 100 m wide and 12 m deep, using a fine discretization of 7500 square elements, to a gradually increasing body force.

1.2.2. Failure mechanism

Fig. 3 shows the contours of sub-accumulated deviatoric plastic strain (from the excavation stage for the examples of sections 1.3 and 1.4), ΔE_{pd} (see Eq. (4)), at the last stable increment of the analysis, for $\phi' = 20°$, illustrating the soil layer failure mechanism which develops tangentially along the interface with the rigid bedrock and extends up to the right mesh boundary.

$$\Delta E_{pd} = \frac{2}{\sqrt{6}}\sqrt{(\Delta\varepsilon_{p1} - \Delta\varepsilon_{p2})^2 + (\Delta\varepsilon_{p2} - \Delta\varepsilon_{p3})^2 + (\Delta\varepsilon_{p3} - \Delta\varepsilon_{p1})^2} \tag{4}$$

where $\Delta\varepsilon_{p1}$, $\Delta\varepsilon_{p2}$, $\Delta\varepsilon_{p3}$ are the sub-incremental principal plastic strains from the excavation stage for the following examples.

The FE analysis resulted in a $k_{lim} = 0.463$, which is slightly higher than the limit equilibrium result of $k_{lim} = 0.444$ based on Eq. (2). This layer mechanism, although theoretically justified, has little physical meaning, as this type of failure has not been observed in post-earthquake reconnaissance investigations. The subsequent examples will demonstrate that this layer mechanism can be mobilised in any type of force based pseudo-static analysis.