Computers and Geotechnics 48 (2013) 72–81

Contents lists available at SciVerse ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Probabilistic parameter estimation and predictive uncertainty based on field measurements for unsaturated soil slope

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ARTICLE INFO

Article history:
Received 21 March 2012
Received in revised form 16 August 2012
Accepted 19 September 2012
Available online 1 November 2012

Keywords:
Rainfall infiltration
Unsaturated soils
Slope failure
Bayesian theory
Markov Chain simulation

ABSTRACT

A key issue in assessment of rainfall-induced slope failure is a reliable evaluation of pore water pressure distribution and its variations during rainstorm, which in turn requires accurate estimation of soil hydraulic parameters. In this study, the uncertainties of soil hydraulic parameters and their effects on slope stability prediction are evaluated, within the Bayesian framework, using the field measured temporal pore-water pressure data. The probabilistic back analysis and parameter uncertainty estimation is conducted using the Markov Chain Monte Carlo simulation. A case study of a natural terrain site is presented to illustrate the proposed method. The 95% total uncertainty bounds for the calibration period are relatively narrow, indicating an overall good performance of the infiltration model for the calibration period. The posterior uncertainty bounds of slope safety factors are much narrower than the prior ones, implying that the reduction of uncertainty in soil hydraulic parameters significantly reduces the uncertainty of slope stability.

1. Introduction

Rainfall-induced slope failures are common in many regions under tropical or subtropical climates. Valid estimation of the reduction of safety factor of a soil slope under rainfall infiltration requires a reliable evaluation of pore water pressure distribution and variations during rainstorm. Numerical investigations [1–3] showed that the unsaturated soil hydraulic properties, i.e., soil–water characteristic curve (SWCC) and unsaturated permeability function, are the most important soil properties to influence the pore-water pressure distribution in soil slopes under rainfall condition. Pore-water pressure distributions in soils are sensitive to relatively small variation of soil hydraulic parameters [4,5]. Therefore, accurate estimation of soil hydraulic parameters is necessary for assessment of slope stability under rainfall.

In geotechnical engineering, direct measurements to determine unsaturated soil hydraulic properties are usually conducted in laboratory using small samples of soils. These tests are time-consuming and difficult, because a long time is required to reach a steady state under certain suctions. However, the lab-measured hydraulic properties may not be directly adopted in field applications, because in situ features including stratification, discontinuities and heterogeneities in the field may not be captured by the small samples of laboratory test [6,7]. In addition, laboratory test results may be affected by sample disturbance. Hence, numerical simulation based on measured unsaturated soil hydraulic parameters from laboratory tests could not match in situ transient pore-water pressure responses very well and significant errors in predicted pore-water pressure profiles and estimated safety factor of the slope may be induced.

On the other hand, pore water pressure measurement for both positive and negative pore pressure (i.e., soil suction) is more frequently used in field monitoring programs of slope stability, besides regular instrumentations for slope stability monitoring such as displacement measurement. Field measured pore water pressure data reflect real response of soils under rainfall infiltration and can provide more representative estimates of in situ soil hydraulic properties [8]. It is hence reasonable to use field observed pore-water pressures to calibrate infiltration models and estimate soil hydraulic parameters for a more accurate evaluation of slope stability under rainfall condition.

Many studies have been used to back analyze soil properties based on field performance of geotechnical structures in literature. Most of these studies focused on soil shear strength parameters [9–12]. Very few studies have been attempted to back analyze soil hydraulic properties. The input information used for the back analysis is often limited to the performance of a geotechnical structure or slope, such as the state of stability and measurement of displacement. Only limited research studies are conducted with the

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0266-352X/$ - see front matter
http://dx.doi.org/10.1016/j.compgeo.2012.09.011

Nomenclature

c	a normalizing constant of probability density function
c'	effective cohesion
d	number of prediction model parameters
Fs	safety factor of slope stability
H	distance from the ground to the slip surface
h_p	pore water pressure head
h_{p1}	prescribed pressure head at the lower boundary
$J()$	jumping distribution or transition kernel of the Markov Chain
k	coefficient of permeability
k_s	saturated coefficient of permeability
L	total depth of the soil
$l(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}})$	likelihood function of $\mathbf{X}$ and σ_e
n	size of simulated outputs of a prediction model
$f(\mathbf{X}, \sigma_e)$	prior distribution of $\mathbf{X}$ and σ_e
$f(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}})$	posterior probability density function of $\mathbf{X}$ and σ_e
q_0	initial surface flux at the time t=0
$q_1(t)$	time-dependent surface flux
R^2	coefficient of determination
R_{stat}	Gelman and Rubin convergence diagnostic
s	scale of inverse chi-square distribution
t	time
u_a	pore air pressure
u_w	pore water pressure
$(u_a - u_w)$	matric suction
$\mathbf{X}$	vector of model input parameters
$\mathbf{X}^*$	vector of candidate point
$\mathbf{Y}$	vector of simulated outputs
$\widehat{\mathbf{Y}}$	vector of observed data
z	elevation or vertical coordinate
α	desaturation rate coefficient of SWCC
β	slope angle
γ_t	total unit weight of soil
γ_w	unit weight of water
$\boldsymbol{\varepsilon}$	vector residual error
θ	volumetric water content
θ_r	residual volumetric water content
θ_s	saturated volumetric water content
λ_n	nth positive root of the characteristic equation
σ_n	total normal stress
σ_e^2	variance of residual error
$(\sigma_n - u_a)$	net normal stress
Φ	matric flux potential
Φ_s	steady state matric flux potential
ϕ'	effective friction angle
ϕ^b	friction angle of matric suction

pore-water pressure distribution or ground water level in soils as inputs [13]. In addition, effect of uncertainties of back estimated hydraulic parameters on prediction uncertainty of slope stability, has yet not been clearly understood. Hence, the objective of this paper is to assess, within the Bayesian framework, the uncertainties of soil hydraulic parameters and their effects on slope stability prediction for slopes under rainfall infiltration, using the field measured temporal pore-water pressure data. The prediction model of slope stability for unsaturated soil slopes under rainfall condition is composed of an analytical solution for one-dimensional rainfall infiltration [14,15] and the slope stability analysis for infinite slope. As the prediction model involves multiple input parameters and is highly nonlinear, the probabilistic back analysis and parameter uncertainty estimation is conducted using the Markov Chain Monte Carlo (MCMC) simulation method [16], with an algorithm entitled Different Evolution Adaptive Metropolis algorithm (DREAM) [17], which has shown good efficiency for highly nonlinear and complex high dimensional problem. A case study of a natural terrain with field measurements of rainfall and pore-water pressures is presented to illustrate the proposed method. The effects of uncertainty reduction of soil hydraulic properties on the predicted pore-water pressures and the safety factor of the slope are illustrated and discussed.

2. Probabilistic parameter estimation within Bayesian framework

Consider the n sized vector of simulated outputs $\mathbf{Y} = \{y_1, \ldots, y_n\}$ of a prediction model G can be written as:

$$\mathbf{Y} = G(\mathbf{X}) \tag{1}$$

where $\mathbf{X} = \{x_1, \ldots, x_d\}$ is the vector of d model parameters.

A common way to assess the model's ability to simulate the underlying system is to compare the vector of model outputs $\mathbf{Y}$ with a vector of n observed data $\widehat{\mathbf{Y}} = \{\widehat{y_1}, \ldots, \widehat{y_n}\}$ by computing the vector of residual errors $\boldsymbol{\varepsilon} = \{\varepsilon_1, \ldots, \varepsilon_n\}$ with

$$\varepsilon_i(\mathbf{X} \mid \widehat{\mathbf{Y}}) = y_i(\mathbf{X}) - \widehat{y}_i \tag{2}$$

The closer to zero are the residuals, the better the model simulates the observed data. However, due to errors in the initial and boundary conditions, structural inadequacies in the model, uncertainties of input model parameters and measurement errors, the residual values of the prediction model are not expected to be equal to zero.

The traditional approach is to force the residual error vector to be as close to zero as possible by tuning the values of the input parameters. However, we can only obtain optimal values of $\mathbf{X} = \{x_1, \ldots, x_d\}$. Based on the Bayes theorem, multiple sources of information such as prior distribution of $\mathbf{X}$ and the observed measurements can be integrated in a systematic way. According to the Bayes theorem, the posterior distribution of $\mathbf{X}$ is proportional to the product of the likelihood function and the prior distribution function [18]. Assuming the residuals in Eq. (2) are mutually independent and Gaussian-distributed with a constant variance, σ_e^2, the likelihood function is

$$l(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_e^2}} \exp\left(-\frac{(y_i(\mathbf{X}) - \widehat{y}_i)^2}{2\sigma_e^2}\right) \tag{3}$$

For simplicity and numerical stability, it is convenient to estimate the logarithm of the likelihood function rather than the likelihood function itself. The log-likelihood of Eq. (3) is:

$$\log\left[l(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}})\right] = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln(\sigma_e^2) - \frac{1}{2}\sigma_e^{-2}\sum_{i=1}^{n}(y_i(\mathbf{X}) - \widehat{y}_i)^2 \tag{4}$$

The posterior probability density function of $\mathbf{X}$ and σ_e can then be written as follows:

$$f(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}}) = c \cdot f(\mathbf{X}, \sigma_e) \cdot l(\mathbf{X}, \sigma_e \mid \widehat{\mathbf{Y}}) \tag{5}$$

where c is a normalizing constant, $f(\mathbf{X}, \sigma_e)$ is the prior distribution of $\mathbf{X}$ and σ_e. With the specification of a prior distribution of parameters, Eq. (5) can be used to calculate the posterior distribution.

Without the information of the observed data $\widehat{\mathbf{Y}} = \{\widehat{y_1}, \ldots, \widehat{y_n}\}$, the predicted model response can be evaluated as: