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A new locally conservative numerical method for two-phase flow in heterogeneous poroelastic media

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ABSTRACT

We construct a new class of locally conservative numerical methods for two-phase immiscible flow in heterogeneous poroelastic media. Within the framework of the so-called iteratively coupled methods and fixed-stress split algorithm we develop mixed finite element methods for the flow and geomechanics subsystems which furnish locally conservative Darcy velocity and transient porosity input fields for the transport problem for the water saturation. Such hyperbolic equation is decomposed within an operator splitting technique based on a predictor–corrector scheme with the predictor step discretized by a higher-order non-oscillatory finite volume central scheme. The proposed scheme adopts an inhomogeneous dual mesh with variable cell size ruled by the local wave speed of propagation to compute numerical fluxes at cell edges. In the limit of small time steps the central scheme gives rise to a semidiscrete formulation for the water saturation capable of incorporating heterogeneous porosity fields and generalized flux functions including the water transport due to the solid phase velocity. Numerical simulations of a water-flooding problem in secondary oil recovery are presented for different realizations of the input random fields (permeability, Young modulus and initial porosity). Comparison between the accuracies of the proposed approach and the traditional one-way coupled hydro-geomechanical formulation are presented. The effects of the cross-correlation between the input random fields and compaction drive mechanism upon finger growth and breakthrough curves are also analyzed. A notable feature of the formulation proposed herein is the accurate prediction of the influence of geomechanical effects upon the unstable movement of the water front, whose evolution is dictated by rock heterogeneity and unfavorable viscosity ratio, without deteriorating the local conservative character of the numerical schemes.

1. Introduction

The study of the coupling between geomechanics and multiphase flows is becoming increasingly important in reservoir engineering as deeper formations are detected and explored. During secondary recovery of hydrocarbon fluid due to forced imbibition of water, changes in pore pressure trigger perturbations in the mechanical equilibrium of the porous medium leading to stress modifications which alter rock properties such as permeability and porosity. Applications are widespread and involve compaction drive mechanism, land subsidence, hydraulic fracturing, stress dependent permeability, pore collapse phenomena, caprock integrity, wellbore instability, casing damage, sand production, strain localization and fault reactivation [76].

The dynamics of hydromechanical coupling in multiphase flow involves highly complex physics associated with different patterns of chaotic fluid mixing and finger growth. Such complex behavior is strongly dictated by heterogeneity in the rock properties acting in conjunction with the unstable mechanism induced by unfavorable viscosity ratio and geomechanical effects [48]. Thermodynamically consistent models exhibiting varying degree of sophistication, with distinct versions of the effective stress principle, have been proposed to describe multiphase and unsaturated flows in shallow formations, where diffusion effects associated with capillary pressure play utmost role in the water movement [2,55]. On the other hand, for problems involving deep extraction of petroleum and gas, where diffusive capillary effects play secondary role, saturation-based formulations are widespread, where this quantity is included in the set of primary unknowns satisfying a hyperbolic equation in the limit of vanishing capillary pressure [28,9,56].

Historically, traditional reservoir models incorporate rock compaction through the well known definition of rock compressibility. Such single concept has a somewhat limited range of application

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restricted to particular idealized loading conditions such as laterally uniform properties (hydrostatic or uniaxial strain), where reservoir and laboratory conditions can be identified [32]. More general loading conditions induced by the adjacent non-pay rocks, which commonly appear in field-scale applications, give rise to complex deviatoric strains which cannot be captured by a single compressibility factor, requiring the use of full tensorial constitutive laws.

In order to properly extend the well-established computational poromechanical models to reservoir conditions, some different degrees of coupling between hydrodynamics and geomechanics have been proposed giving rise to distinct formulations commonly referred to as fully coupled and sequential algorithm approaches [69,66,20,64]. Within this latter group one can highlight iteratively coupled methods, explicit coupled and loosely coupled schemes [66,20,69,63]. Stability and convergence analysis of drained and undrained split sequential algorithms have been discussed in [39]. In spite of its well established applications in soil consolidation analysis, the numerics of fully-coupled algorithms, where stress and flow equations are solved simultaneously, entails a large number of degrees-of-freedom imposing great computational challenges, large time run and consequently may become unfeasible [38]. Moreover, the simultaneous solution strategy enforces the same time-scale for flow and geomechanics which precludes the development of algorithms capable of exploring different time-scales to reduce the computational cost. Within partitioned approaches the system is solved in a staggered fashion over the same time interval with periodic transfers based on fixed strain of fixed stress algorithm (see [65], for elasticity and [38] for non-linear poromechanics).

Such form of split has been implemented in the context of iteratively coupled algorithms which tackle flow and geomechanics modules in a iterative fashion showing unconditional stability and high accuracy. Other procedures include the explicit coupling wherein the geomechanics module is updated at selected time steps [20] and more simplified algorithm, commonly referred to as one-way coupling formulation, has been widely adopted based on implementation of interfaces between flow and geomechanics simulators with particular forms of the transfer functions. In this context flow affects geomechanics through a source term involving the pore pressure gradient whereas in the reverse direction geomechanical coupling is solely manifested through the compressibility concept [63,4].

In spite of the recent advances on improving the performance of the different degrees of coupling in single phase flows [63,46], further improvements are necessary to extend their application to multiphase flows, where additional hyperbolic transport equations for the phase saturations are included in the underlying model, and to include multiscale heterogeneity in the formation properties at the field-scale [48,26,24]. Natural reservoirs are spatially heterogeneous with properties varying in an erratic manner exhibiting spatial fluctuations over a vast range of length scales [30]. The appearance of inhomogeneities over multiple scales suggests that samples tested in the laboratory are not representative of the in situ behavior. Multiscale heterogeneities give rise to uncertainty and scaling issues in the measurements of the coefficients which become random space functions. Erratic variations in the permeability distribution and elastic constants produce fluctuations in the seepage velocity and effective stresses leading to the initiation of fingering instability which impacts a variety of hydrocarbon recovery processes by spreading the uncertain poromechanical unknowns around their mean values. The incomplete knowledge of data requires the construction of stochastic geomechanical models rather than deterministic theories to tackle the problem [30,19,60].

One of the most challenging issues in computational geomechanics is the development of new classes of locally conservative computational schemes capable of capturing in an accurate fashion the effects of spatial variability in the formation properties by handling highly heterogeneous coefficients with complex spatial distributions while preserving local conservation properties. The conventional Galerkin method with continuum Lagrangian interpolation lacks capability to capture the complex fine-scale variability of the unknowns, as it enforces conservation of mass only approximately [50]. To overcome this difficulty the development of locally conservative methods represents an active research area [22]. For instance, a well established family of mixed finite elements, finite volumes and discontinuous Galerkin procedures has been developed to enforce local conservation properties [44,59,31]. In particular mixed methods, based on simultaneous approximation of potentials and fluxes, were designed for approximating the Darcy's velocity using the Raviart–Thomas spaces [11]. Unlike classical Lagrangian interpolation, these spaces mimic the local physics by approximating heterogeneous velocity fields using continuum interpolation across element edges for the normal flux while maintaining the jumps in the tangential component under high contrast in permeability of adjacent geological blocks.

In fact the resemblance between mixed methods and finite volume/finite difference techniques has been widely discussed in the literature (see e.g. [61,74]). More precisely the utilization of the lowest order Raviart–Thomas spaces supplemented by a special quadrature integration rule reproduces a multipoint-flux centered finite difference scheme where sub-edge fluxes are introduced leading to larger stencils [73,1,70,6]. Also mixed hybrid finite element methods with introduction of Lagrange multipliers at the element edges have also been widely adopted and can be envisioned as face centered finite volume methods (see e.g. [10]).

More recently applications of mixed methods have been extended to single phase flow in poroelastic media with particular emphasis placed on improving the numerical approximation of the hydrodynamics subsystem [57,49,37,25]. Further, a primal mixed formulation for the elastic part of the model has been proposed in Tehonkova et al. [68]. The issue of interpolating symmetric tensors through tensorial products of Raviart–Thomas spaces is still challenging and partially overcome through the use of methods that impose weak symmetry requiring additional degrees of freedom [3].

Unlike hydrid and cell centered finite volume methods, based on a global equation for the scalar and local post-processings for the Darcy velocity, here we exploit an alternative implementation of the mixed formulation for parabolic problems, where compressibility effects are explored to locally eliminate pressure discontinuous degrees of freedom resulting in a formulation solely in terms of velocity. In contrast to the aforementioned pressure formulations, the Neumann boundary condition of imposed volumetric flow rate becomes essential-type of boundary condition which is capable of capturing more accurately flow across the Neumann segments of the inlet boundary.

For multiphase flow with negligible capillary effects the poromechanics is coupled to hyperbolic conservation laws which govern the evolution of phase saturations and whose construction of locally conservative numerical solutions is essential to resolve correctly the speed of the saturation wave front. In order to preserve the local conservation property in the hyperbolic equation, numerical methods used to compute Darcy's velocity and transient input porosity shall also be locally mass-conservative. Numerical schemes not fulfilling this requirement may lead to unrealistic finger growth and erroneous breakthrough curves [21].

Some preliminary attempts toward the development of locally conservative methods for two-phase flow in heterogeneous poroelastic media have been proposed in Mendes et al. [48]. Within the context of the fully-coupled formulation based on a Petrov–Galerkin post-processing approach to compute the velocity of the mixture in conjunction with the finite volume method based on the non-oscillatory NT central scheme proposed by Nessyahu and