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Two-dimensional probabilistic infiltration analysis with a spatially varying permeability function

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A B S T R A C T

The permeability function for a soil may change spatially due to uncertainties in soil fabric. The main objective of this paper is to investigate how the spatial variability of permeability function propagates to the variability of the pore-water pressures and groundwater table in a slope as well as the stability of the slope. A random field analysis method is explored by assigning discrete random values to a 2D space and controlling the density of random field grid to improve the calculation accuracy. Sequences of random numbers are generated using fast Fourier transform. In a given heterogeneous slope subject to steady-state rainfall infiltration, a parametric study shows that the matric suctions are 0.5–1.25 times those in a homogeneous slope when the correlation length of log-permeability varies from 0.4 to 50 times the slope height. The groundwater table is no longer unique with a spatially variable permeability function. There exists a critical correlation length approximately five times the slope height at which the change in the groundwater table is maximal and the mean factor of safety is minimal. The mean factor of safety of the heterogeneous slopes is smaller than that of a homogenous slope with mean input parameters. The spatial variability of soil influences the range of the calculated factor of safety significantly but does not influence the mean factor of safety substantially.

1. Introduction

It has been widely recognized that matric suction (or negative pore-water pressure when the air pressure is zero) in a soil slope contributes to the soil shear strength and thus the slope stability [1]. As rainfall infiltrates into the soil, the pore-water pressure becomes less negative or even positive. As a result, the soil shear strength decreases, which may finally lead to slope failure. Numerical models for infiltration and stability analysis of both saturated and unsaturated slopes have been developed [2–4].

Most geotechnical analyses adopt deterministic approaches based on the assumption that soil properties are not random but deterministic values. Nevertheless, even within a given soil mass, hydraulic and shear strength properties often vary significantly from point to point as a result of depositional and post-depositional processes [5,6]. Probabilistic methods have been introduced to account for the uncertainty and spatial variability of soil parameters. Geotechnical parameters, such as soil shear strength, compressibility and hydraulic parameters, are often taken as either normal or lognormal variants [7]. Recent studies have considered the spatial fluctuation of a parameter by using random field theory [8–10]. In a random field, the variables exhibit autocorrelation, which is a tendency for soil properties at one point to be correlated to soil properties at nearby points. A classic paper that introduces the spatial correlation concept was published by Vanmarcke [11].

Griffiths et al. [12] studied the effects of spatial variability of soil strength parameters on slope stability in two dimensions. Santoso et al. [13] performed one-dimensional (1D) infiltration analysis in an infinite slope considering the saturated permeability as a random field. Gui et al. [14] investigated heterogeneous soil hydraulic conductivity and its influence on the phreatic surface in a dam under transient seepage conditions and the reliability index of the dam. Cho [15] presented a probabilistic analysis of seepage in layered soil and considered the spatial variability of hydraulic conductivity. Previously, variations in pore-water pressures and safety factors have been analyzed using the random finite element method proposed by Griffiths and Fenton [9] in which the random field cell and finite element coincide with each other. Griffiths et al. [12] commented that the random field cell and finite element can have different sizes. Many different random field generation algorithms are available among which the following are perhaps the most common [16]: the local average subdivision method (LAS), the moving average method; discrete Fourier transform; covariance matrix decomposition; fast Fourier transform (FFT); and the

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turning-band method (TBA). Particularly, the local average subdivision method proposed by Fenton and Griffith [9,16–18] has been frequently adopted in geotechnical analysis. Griffith and Fenton [16,19] estimated the mean and variance fields using FFT, TBA and LAS. These methods all yield very good results with expected mean and variance quantiles. FFT is computationally efficient and can handle spatially heterogeneous fields more easily [20].

The permeability function for a soil refers to the relation between the coefficient of permeability of the soil and the matric suction. It may change spatially due to uncertainties in soil fabric. It is of great significance to perform probabilistic infiltration and stability analysis of slopes considering the permeability function as a random field. Few attempts have been made to study the variations in groundwater table in random field analysis. The statistical response of pore-water pressures in two-dimensional (2D) unsaturated heterogeneous slopes has not been sufficiently investigated. Also the critical hydraulic conditions that may lead to failure of heterogeneous slopes are not well known.

The objectives of this paper are: (1) to explore a random field analysis method by assigning discrete random values to a 2D space using a FFT method and controlling the density of the random field grid to improve calculation accuracy; (2) to investigate how the variability of permeability function propagates to the variability of hydraulic conditions (i.e. pore-water pressure and groundwater table) induced by steady rainfall infiltration; (3) further to conduct stability analysis based on the seepage analysis results. The uncertainty of the permeability function is characterized by the spatial variability of the coefficient of permeability (k_s).

2. Methodology of random field generation

2.1. Correlation structure of 2D random field

The generation of a random field normally requires a mean (μ), a standard deviation (σ) or coefficient of variation (CoV) and a correlation structure with a given correlation length (d) [21]. Based on field measurements, previous studies have shown that k_s can be modeled as a lognormal random field [22], with a mean of μ_{k_s} and a standard deviation of σ_{k_s}. Thus, $\ln k_s$ follows a normal distribution with a mean of $\mu_{\ln k_s}$ and a variance of $\sigma^2_{\ln k_s}$, where

$$\sigma^2_{\ln k_s} = \ln\left(1 + \sigma^2_{k_s}/\mu^2_{k_s}\right) \quad (1a)$$

$$\mu_{\ln k_s} = \ln(\mu_{k_s}) - \frac{1}{2}\sigma^2_{\ln k_s} \quad (1b)$$

In Eq. (1), $\sigma^2_{\ln k_s}$ is dimensionless. Within the correlation length, values of log-permeability at two points show correlation; the tendency of which decreases with the distance between them. Values at any two points separated by distances beyond the correlation length have little correlation and behave independently. The k_s random field is assumed to be statistically stationary. This is reasonable when a soil profile contains similar deposit. Values of k_s at every location within the soil profile follow the same distribution with the same statistical parameters. In this study, not only a stationary random field is considered, but also a common spherical correlation structure is assumed, which quantitatively describes the correlation between two observations in an isotropic field where the correlation lengths in the horizontal and vertical directions are the same and can be expressed as:

$$\rho = 1 - 1.5(\tau/d) + 0.5(\tau/d)^3, \quad \tau \leqslant d \quad (2a)$$

$$\rho = 0, \quad \tau > d \quad (2b)$$

where ρ is the correlation coefficient of two observations; τ is the distance between two observations; d is the correlation length.

2.2. Random field generation using Fast Fourier Transformation

In order to create a random field of a soil parameter, a rectangular random field is first generated, which provides a pool to draw the random values of saturated permeability parameter (k_s) from. A random field is a list of discrete random values, which can be mapped onto space by interpolation. The values of a random variable at one point of the field are correlated with values within a distance from the point. This paper applies a fast Fourier transform code modified from Kozintsev [23] to explore its calculation efficiency and performance, although various random number generators are available including the Mersenne-Twister method [24]. A sequence of data is first generated with parameters represented by zero mean, unit variance, and a spherical correlation structure in C language. Iterations are further carried out in Monte Carlo Simulation by analyzing 1000 realizations, each representing an independent random number sequence with 100×40 numbers.

The random sample of the correlated standard normal variable is assigned to a 2D domain from the bottom left corner to the top right corner following the arrows in Fig. 1a. Thus, Gaussian random fields with point values at regular grid points are produced and a lognormally distributed random field is given by

$$\ln k_s(x_i, y_i) = \mu_{\ln k_s} + \sigma_{\ln k_s} z_i(x_i, y_i) \quad (3)$$

$$k_s(x_i, y_i) = \exp\{\ln k_s(x_i, y_i)\} \quad (4)$$

where (x_i, y_i) is the spatial position of random number z_i (referring to the intersection point in Fig. 1), at which a sample of k_s is desired; z is the vector of the correlated random numbers, z_i. Fig. 2 presents sample random fields for z, $\ln k_s$ and k_s that are mapped using bilinear interpolation. z and $\ln k_s$ are linearly related and $\ln k_s$ is an exponential function of k_s. Then the random field is truncated to the geometry of the slope, shown in Fig. 3. In this study, the mapping procedure is implemented in the seepage analysis, which will be described later in this paper.

2.3. Effect of density of random field grid

The density of a random field grid can be controlled in this study and it can influence the distribution of random values to some extent. Fig. 1a presents a coarse grid of random field and Fig. 1b gives a fine grid. The amount of random numbers generated by the FFT method can vary, following which the density of grid is changed. For instance, the use of 100×40 cells requires 4000 data, while the use of 200×80 cells requires 16,000 data as shown in Fig. 1. The density of grid used in this study depends on calculation efficiency and desired accuracy. The effect of the grid density on the pore-water pressure profile will be illustrated and discussed later.

3. Deterministic models for infiltration and stability analysis

3.1. Infiltration analysis under rainfall condition

3.1.1. Governing equation

In this paper, we consider steady-state seepage through partially saturated media with spatially distributed permeability functions. The soil permeability influences the flow rate according to Darcy's law and the spatial variation of the saturated permeability, k_s, may change the water flow paths and the pore-water pressure distribution. Infiltration analysis in an unsaturated soil has been performed in previous studies [25–27]. Common computer programs for numerical modelling of seepage and infiltration in unsaturated slopes include Seep/W [28], SVflux [29], Flow3D [30] and FEMWATER [31].