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Evaluation of models for laterally loaded piles

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A R T I C L E I N F O

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A B S T R A C T

It is common in the analysis of piles under lateral loads to use a model of a beam on elastic foundation, or a finite element model with the pile represented by a one dimensional beam–column with its axis coinciding with the central line of the finite element mesh. In both cases the lateral stiffness of the pile itself, as a structural element, is a function of the product of its Young's modulus of elasticity by the moment of inertia of the cross section (EI). For solid piles the moment of inertia is directly related to the radius but this is not the case when dealing with hollow piles where the value of the radius corresponding to a given moment of inertia is not unique. Both of the above models ignore the effect of the value of the radius of the soil cavity occupied by the pile. In this work a more accurate model of the pile with the soil around it represented. A consistent boundary matrix valid for static and dynamic analyses is used to evaluate the accuracy of the results provided by the model of a beam on elastic foundation. In addition, a 1D model of the pile is analyzed with finite elements for the soil. This analysis considers a fixed value of the product EI, but a variable radius in order to illustrate the importance of the radial dimension. Results are obtained for a pile fixed at the bottom, but long enough so that the top boundary conditions do not affect the results and for a shorter floating pile were the shear and moment at the bottom resulting from the underlying soil would not be zero. For the beam on elastic foundation model, the top of the pile was assumed to be fixed.

1. Introduction

A model commonly used for the analysis of laterally loaded piles is that of a beam on an elastic (Winkler) foundation. This model has been used for linear and nonlinear (*p*–*y* curves) static analyses. Novak [1] and Novak and Nogami [2] used it for dynamic analyses with the springs and dashpots functions of frequency obtained from Baranov's equations [3] Flores Barrones and Whitman [4], Dobry and O'Rourke [5], Mylonakis [6] and Nikolaou et al. [7] used a regular Winkler model to study seismic forces in piles caused by the wave passage (kinematic interaction). De Sanctis et al. [8] used a finite element model for the soil but they did not indicate whether the pile had been represented by a linear member (its centroidal axis) placed along the centerline of the mesh or whether they had used solid 3D elements for a solid pile or shell elements for a hollow one.

A more accurate solution, incorporating the coupling between forces and displacements along the pile, had been presented by Poulos [9,10] considering the pile as a very thin rectangular strip of width D (pile diameter for a circular pile) and using Mindlin's equation [11] for a point load in the interior of an elastic half space (integrated over a rectangular area) to obtain the displacements at various points along the pile due to a force applied at one level. These results depended on whether one used the displacements at the centerline or at the edges of the rectangular area. It is interesting to notice that a very similar, and in fact more accurate, approach had been proposed a few years before by Penzien et al. [12] integrating Mindlin's equation over a cylindrical area rather than over a plane rectangle. In this case the displacement would be a function of the azimuthal angle and the authors used an average value over the circumference. Surprisingly this solution did not receive as much recognition as Poulos'. An even better solution was presented by Blaney et al. [13] using the consistent boundary matrix developed by Kausel [14] to reproduce the soil and enforcing compatibility of horizontal and vertical displacements between the soil and the external surface of the pile.

Sanchez Salinero [15] conducted an extensive set of comparisons between the results provided by Poulos' [9,10], Penzien et al.'s [12], Novak and Nogami [2] and Blaney et al. [13] for the static case (using a dimensionless frequency of 0.3 for Baranov's equations [3]) and those of Novak and Nogami [2] and Blaney et al. [13] for the dynamic case (as a function of the frequency of the applied force). He considered only solid piles and he concluded that the agreement between the more accurate solutions and those provided by the Winkler foundation model were reasonable for

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values of the pile's radius of 0.5–1.0 m, and values of the ratio of the moduli of elasticity of the pile and the soil reasonably large.

As part of a study of the reliability of pile foundations for offshore wind towers, where it is common to use large diameter hollow monopiles, Markfedri [16] compared more recently the predictions provided by a nonlinear finite element model with those obtained with Winkler models and the traditional *p*–*y* curves in the nonlinear case. As a first step, to calibrate the models to be used for the nonlinear analyses, she considered three linear elastic models: one corresponding to a beam on a Winkler (elastic) foundation, a second one using a three dimensional representation of the soil and the pile with solid and shell finite elements respectively, and a third one where the finite element mesh covered the complete region of interest, without a cavity, and the pile was modeled as a one dimensional beam with its axis coinciding with the central line of the finite element soil model enforcing only compatibility of horizontal displacements. The first model could include both horizontal and rotational springs distributed along the axis of the pile. With rotational springs the model will be affected by the value of the radius (albeit only slightly), whereas when considering only horizontal springs the results will be independent of the radius and only function of the moment of inertia and Young's modulus of the material. The same will be true for the third model. Markfedri [16] found that the results obtained with the Winkler model using the values of the lateral springs recommended by Novak [1] and based on Baranov's expressions [3] for a dimensionless frequency of 0.3 (the values are 0 for the static case, since the solution is two dimensional) agreed reasonably well with those of the more accurate finite element model for the large diameter hollow pile considered but that the difference would increase as the radius changed. The results provided by the finite element model with the one dimensional beam were way off those provided by the more accurate solutions, while the results using the 3D finite element model were in excellent agreement with those provided by Blaney et al.'s [13] approach.

The objective of the present study was to investigate further the effect of variations on the value of the radius of the pile for a fixed value of the stiffness term EI with special emphasis on hollow piles. Four different models are considered: an accurate model where the soil surrounding the pile is reproduced by a consistent boundary matrix whose terms will depend on the radius, and where compatibility is enforced between horizontal and vertical displacements of the pile and the soil along their interface (i.e. Blaney's model); a Winkler model with only horizontal springs; a Winkler model with both horizontal and rotational springs; and the finite element model with a zero radius cavity (pile represented as a line). Fig. 1a) represent schematically the first case (i.e. boundary matrix); Fig. 1b) is related to the Winkler solution and Fig. 1c) is associated with finite element solution. The solution for the two Winkler models were obtained directly from the expressions presented hereafter. The results obtained and compared are the flexibility coefficients at the pile head representing the displacement and rotation due to a unit horizontal force applied at the top of the pile 'f_{xx}' and 'f_{xr}'; and the displacement and rotation corresponding to a unit moment 'f_{rx}' (='f_{xr}') and 'f_{rr}'.

2. Formulation

The more refined solution was obtained representing the soil around the pile by a consistent boundary matrix as defined by Kausel [14] and implemented by Blaney et al. [13] for the dynamic analysis of single piles. This is a dynamic solution for steady-state harmonic vibrations but the static case is just a particular case with the frequency equal to zero (or very small). The same approach was used by Sanchez Salinero [15]. The formulation was implemented in a computer program written for this purpose.

For the beam on elastic foundation one can obtain closed form analytical solutions if one assumes that the stiffness of the Winkler foundation is constant with depth (a uniform soil layer). For the general case with both horizontal and rocking springs 'k_x' and 'k_r' distributed along the pile the differential equation is [15]:

$$EI\,v^{iv} - k_r v'' + k_x v = 0 \tag{1}$$

where E is the modulus of Elasticity of the pile, I the moment of inertia of pile cross section, and v is the horizontal displacement.k_x and k_r have units of force per length square and force per unit of length with boundary conditions assuming the pile hinged at the bottom:

$$v''_{(0)} = \frac{M}{EI} \tag{2}$$

$$v'''_{(0)} - \frac{k_r}{EI} v'_{(0)} = \frac{H}{EI} \tag{3}$$

and at the bottom

$$v_{(L)} = v''_{(L)} = 0 \quad \text{for a hinged tip} \tag{4}$$

$$v_{(L)} = v'_{(L)} = v''_{(L)} = 0 \quad \text{for a fixed tip} \tag{5}$$

where M is the Moment, H the Horizontal load and L is the pile length.Calling:

$$\alpha^4 = \frac{k_x}{4EI} \tag{6}$$

$$\beta^2 = \frac{k_r}{2EI} \tag{7}$$

$$\lambda^2 = \alpha^2 + \beta^2 \tag{8}$$

$$\mu^2 = \alpha^2 - \beta^2 \tag{9}$$

where α and β are measured in units of 1/length (they can be interpreted as wavenumbers in the pile).

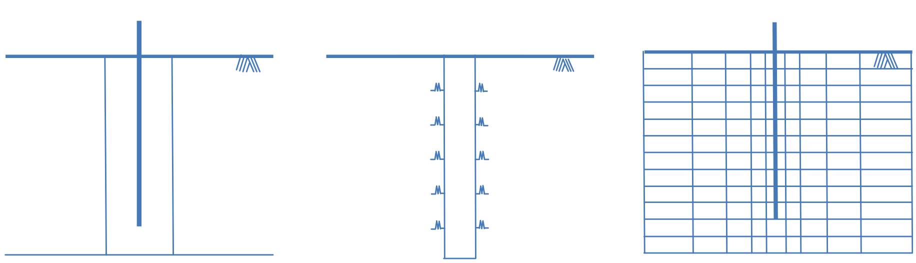

Fig. 1. Schematic representation of the main different cases studied: (a) boundary matrix; (b) Winkler solution; and (c) finite element method.

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