J. Non-Newtonian Fluid Mech. 166 (2011) 404–412

Contents lists available at ScienceDirect

Journal of Non-Newtonian Fluid Mechanics

journal homepage: www.elsevier.com/locate/jnnfm

Lattice Boltzmann simulations of viscoplastic fluid flows through complex flow channels

Mitsuhiro Ohta [a,*], Tatsuya Nakamura [a], Yutaka Yoshida [a], Yosuke Matsukuma [b]

[a] Division of Applied Sciences, Graduate School of Engineering, Muroran Institute of Technology, 27-1 Mizumoto-cho, Muroran, Hokkaido 050-8585, Japan
[b] Department of Chemical Engineering, Graduate School of Engineering, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka 819-0395, Japan

ARTICLE INFO

Article history:
Received 1 September 2010
Received in revised form 7 January 2011
Accepted 26 January 2011
Available online 1 March 2011

Keywords:
Numerical simulation
Lattice Boltzmann method
Viscoplastic fluids
Complex flow channels

ABSTRACT

We present the results of lattice Boltzmann (LB) simulations for the planar-flow of viscoplastic fluids through complex flow channels. In this study, the Bingham and Casson model fluids are covered as viscoplastic fluid. The Papanastasiou (modified Bingham) model and the modified Casson model are employed in our LB simulations. The Bingham number is an essential physical parameter when considering viscoplastic fluid flows and the modified Bingham number is proposed for modified viscoplastic models. When the value of the modified Bingham number agrees with that of the "normal" Bingham number, viscoplastic fluid flows formulated by modified viscoplastic models strictly reproduce the flow behavior of the ideal viscoplastic fluids. LB simulations are extensively performed for viscoplastic fluid flows through complex flow channels with rectangular and circular obstacles. It is shown that the LB method (LBM) allows us to successfully compute the flow behavior of viscoplastic fluids in various complicated-flow channels with rectangular and circular obstacles. For even low *Re* and high *Bn* numbers corresponding to plastic-property dominant condition, it is clearly manifested that the viscosity for both the viscoplastic fluids is largely decreased around solid obstacles. Also, it is shown that the viscosity profile is quite different between both the viscoplastic fluids due to the inherent nature of the models. The viscosity of the Bingham fluid sharply drops down close to the plastic viscosity, whereas the viscosity of the Casson fluid does not rapidly fall. From this study, it is demonstrated that the LBM can be also an effective methodology for computing viscoplastic fluid flows through complex channels including circular obstacles.

1. Introduction

The understanding of non-Newtonian fluid flows through porous media is of principal significance in various science and engineering processes. A few examples are non-Newtonian flows in packed beds, non-Newtonian flows through a fiber material, filtration and purification processes in chemical, petroleum and polymer engineering fields. The exploration of non-Newtonian fluid flows is a far from easy issue because of the complex rheological properties. Hence, a detailed understanding of non-Newtonian fluid flows requires consideration from various aspects and this type of research is an interdisciplinary subject. In terms of fluid dynamics, a key factor to exploring structures and mechanisms of non-Newtonian fluids is to perceive a local profile of non-Newtonian properties corresponding to the shear-rate. An approach based on computational fluid dynamics (CFD) can be effective for revealing the local non-Newtonian properties because important physical information specific to non-Newtonian fluids can be locally estimated and we can reveal the role and state of non-Newtonian properties by numerical simulation.

Over the last two decades, the lattice Boltzmann method (LBM) [1–4] has been aggressively developed as an alternative approach to common numerical methods based on the direct discretization of the incompressible Navier–Stokes equations. Important advantages of the LBM are simplicity of programming, parallelism potential of algorithm, easy grid generation, straightforward implementation (i.e., streaming and collision), and the LBM can be also easily applied to complex geometries. An essential advantage of the LBM is that it is suitable for solving flow problems involving complicated boundary geometries. Now the LBM has become an established numerical technique for simulating single-phase and multiphase fluid flows in complex geometries [2].

The first attempt applying the LBM to simulation of non-Newtonian fluid flows was carried out by Aharonov and Rothman [5]. They demonstrated the applicability of the LBM to power–law model [6] fluids. Results of LB simulations for the power–law model fluids have been given by a number of researchers [7–15] since the work of Aharonov and Rothman [5]. Although the power

* Corresponding author. Tel.: +81 143 46 5767; fax: +81 143 46 5767.
E-mail address: mohta@mmm.muroran-it.ac.jp (M. Ohta).

0377-0257/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.jnnfm.2011.01.011

law model has the disadvantage which the viscosity of the power law model can be potentially infinite at low shear-rate regions, these studies would be very valuable for understanding the most essential effect of shear-thinning fluids. Meanwhile, viscosity models including the zero shear-rate viscosity η_0 and the infinite shear-rate viscosity η_∞ are much more practical because shear-thinning and shear-thickening fluids generally have Newtonian plateaus with η_0 and η_∞.

Kehrwald [16] carried out the LB simulation for shear-thinning fluids using the Cross model [17] with η_0 and η_∞. From a mathematical standpoint, the Carreau–Yasuda model [18] is best suited to express the viscosity property of shear-thinning and shear-thickening fluids. In the computational study, the choice of the Carreau model [19] is reasonable for shear-thinning and shear-thickening fluids. LB simulations for the Carreau model fluids have been presented by Yoshino et al. [20] and Malaspinas et al. [21]. Recently, with a focus on human blood flow, non-Newtonian simulations using the LBM have also been considered by a number of researchers [22–28]. The Carreau–Yasuda model [18] and the Casson model [29] are commonly used to simulate blood flows. As examples of the applicability of the LBM to other non-Newtonian fluids, we can find some computational results for viscoplastic fluid flows. Ginzburg and Steiner [30] considered the LB simulation of viscoplastic fluid flows with the free surface. In their study, the Papanastasiou model [31] was used for viscoplastic liquids. Similarly, using the Papanastasiou model [31], Wang and Ho [32] presented the results of LB simulation for the flow of the Bingham fluid [33] in an abrupt expansion planar channel. Vikhansky [34] proposed an improved LBM for simulating the flow of the Bingham fluid and showed LB simulations for Bingham fluid flows through channels including a cylindrical obstacle. In the study of Vikhansky [34], the "ordinal" Bingham model [33] was employed. We note that Barnes and Walters [35] demonstrated that the yield stress concept is an idealization and that no yield stress exists, but the issue of the existence of yield stress has been disputed [35–40].

As just described, in the past decade or so, the application of the LBM to non-Newtonian fluid flows has been intensified and much progress has been reported in simulating non-Newtonian fluid flows involving complex boundaries. Nevertheless, there remain several problems to be considered. The present study focuses on the planar-flow of viscoplastic fluids through various complex flow channels including rectangular and circular obstacles. The Bingham and Casson models are adopted for viscoplastic fluids, and the flow behavior for such viscoplastic fluids are considered by computational results obtained by using the LB simulation. Major novelties of our study different from previous LB simulations for viscolastic fluids [30,32,34] are (1) to target two kinds of viscoplastic fluid which are described by modified Bingham and Casson models based on the Papanastasiou-type expression, and (2) to consider the flow of both the viscoplastic fluids through complex channels including circular obstacles. Finally, we show the effectiveness of the LB simulation for computing viscoplastic fluid flows in complex channels and discuss flow situations depending on both the models when both viscoplastic fluids flow through complex channels.

2. Numerical method

2.1. Lattice Boltzmann method

In this study, we cover incompressible fluid flows and a nine-velocity model on a two-dimensional lattice (D2Q9) as shown in Fig. 1. Here D is the spatial dimension and Q refers to the number of different velocities at a node. In the LBM, the fluid behavior is described by the particle distribution function $f_i(\mathbf{x}, t)$ giving the probability that a fictitious fluid particle with velocity $\boldsymbol{e}_i$ enters the lattice site $\mathbf{x}$ at time t. Here, the subscript "i" expresses the number of lattice links and $i = 0$ corresponds to the particle at rest residing in the center (see Fig. 1). The evolution of the particle distribution function on the lattice resulting from the collision processes and the particle propagation is governed by the discrete Boltzmann equation:

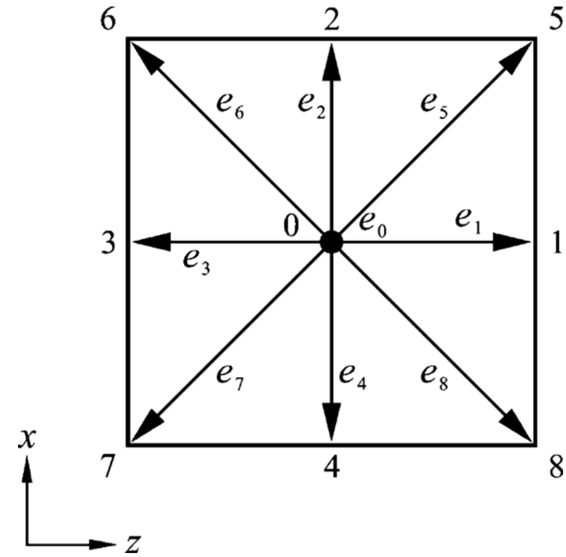

Fig. 1. Direction system and discrete velocity vectors in the D2Q9 model. Direction "0" corresponds to a rest state.

$$f_i(\mathbf{x} + \mathbf{e}_i\Delta t, t + \Delta t) - f_i(\mathbf{x}, t) = \Omega_i(\mathbf{x}, t) \quad (i = 0, 1, \ldots, 8), \tag{1}$$

where Δt is the time step and Ω_i is the collision operator which accounts for the rate of change in the distribution function due to the collisions. In our LB computations, the Bhatnagar–Gross–Krook (BGK) model [41,42], which utilizes a simple relaxation for complicated collisions [43], is used for the collision operator.

$$\Omega_i(\mathbf{x}, t) = -\frac{1}{\lambda}[f_i(\mathbf{x}, t) - f_i^{(\mathrm{eq})}(\mathbf{x}, t)] \quad (i = 0, 1, \ldots, 8), \tag{2}$$

where λ is the non-dimensional relaxation time and is related to the kinematic viscosity ν by

$$\nu = c_S^2 \Delta t \left(\lambda - \frac{1}{2}\right). \tag{3}$$

Here c_S is the sound speed expressed by $c_S = \Delta x/3^{0.5}\Delta t = c/3^{0.5}$ (c is the particle speed and Δx is the lattice spacing). The constant c can be freely chosen and we take $c = \Delta x = \Delta t = 1$ in this study. All physical quantities are rendered dimensionless using Δx and Δt as the characteristic scales.

In our computations, $\lambda(\mathbf{x}, t)$ is calculated at each node using Eq. (4) because $\nu(\mathbf{x}, t)$ is locally changed as a function of the shear-rate $\dot{\gamma}$:

$$\lambda(\mathbf{x}, t) = 3\nu(\mathbf{x}, t) + \frac{1}{2} = \frac{3\eta(\mathbf{x}, t)}{\rho(\mathbf{x}, t)} + \frac{1}{2}. \tag{4}$$

Here $\eta(\mathbf{x}, t)$ is the apparent viscosity of viscoplastic fluids which will be explained in the next section and $\rho(\mathbf{x}, t)$ is the density. Also, $f_i^{(\mathrm{eq})}(\mathbf{x}, t)$ in Eq. (2) is the corresponding equilibrium distribution function. The equilibrium form for the D2Q9 lattice is given as follows:

$$f_i^{(\mathrm{eq})}(\mathbf{x}, t) = w_i\rho(\mathbf{x}, t)\left[1 + \frac{1}{c_S^2}(\mathbf{e}_i \cdot \mathbf{u}(\mathbf{x}, t)) + \frac{1}{2c_S^4}(\mathbf{e}_i \cdot \mathbf{u}(\mathbf{x}, t))^2 - \frac{1}{2c_S^2}(\mathbf{u}(\mathbf{x}, t) \cdot \mathbf{u}(\mathbf{x}, t))\right], \tag{5}$$