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Purely elastic flow asymmetries in flow-focusing devices

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ABSTRACT

The flow of a viscoelastic fluid through a microfluidic flow-focusing device is investigated numerically with a finite-volume code using the upper-convected Maxwell (UCM) and Phan-Thien–Tanner (PTT) models. The conceived device is shaped much like a conventional planar "cross-slot" except for comprising three inlets and one exit arm. Strong viscoelastic effects are observed as a consequence of the high deformation rates. In fact, purely elastic instabilities that are entirely absent in the corresponding Newtonian fluid flow are seen to occur as the Deborah number (*De*) is increased above a critical threshold. From two-dimensional numerical simulations we are able to distinguish two types of instability, one in which the flow becomes asymmetric but remains steady, and a subsequent instability at higher *De* in which the flow becomes unsteady, oscillating in time. For the UCM model, the effects of the geometric parameters of the device (e.g. the relative width of the entrance branches, *WR*) and of the ratio of inlet average velocities (*VR*) on the onset of asymmetry are systematically examined. We observe that for high velocity ratios, the critical Deborah number is independent of *VR* (e.g. $De_c \approx 0.33$ for $WR = 1$), but depends non-monotonically on the relative width of the entrance branches. Using the PTT model we are able to demonstrate that the extensional viscosity and the corresponding very large stresses are decisive for the onset of the steady-flow asymmetry.

1. Introduction

While Newtonian fluid flows may exhibit inertial instabilities as the Reynolds number (*Re*) increases, viscoelastic fluid flows may develop elastic instabilities as the Deborah number (*De*) increases for arbitrarily small Reynolds numbers. The latter are the subject of extensive reviews (e.g. [1,2]). Shaqfeh [2] focused on purely elastic instabilities in viscometric flows in rheometry devices, which include Taylor–Couette [3,4], parallel plate flow [5–7] and cone-and-plate flows [7,8]. These instabilities received extensive interest since they can be found in a wide variety of applications including polymer processing, lubrication and coating, and can make the nominally "viscometric" flow unsuitable for rheological measurements. The underlying mechanism that leads to such instabilities is associated with the large normal-stress differences, which depend non-linearly on flow velocity and streamline curvature. In particular, it has been shown that the tensile stress along the streamlines and the local curvature of the flow play a decisive role on the onset of purely elastic instabilities [9,10]. McKinley et al. [10] define a dimensionless criterion that must be exceeded for purely elastic instability to be observed.

The advent of microfluidics has promoted a renewed interest in purely elastic flow instabilities, which occur in the absence of inertial forces when elastic forces are very strong. The small length scales characteristic of microfluidics enable the generation of flows with high deformation rates while keeping the Reynolds number small. Conditions in microfluidic devices result in the ability to promote strong viscoelastic effects, which are not masked by fluid inertia, in dilute solutions that would otherwise exhibit Newtonian-like behavior at the equivalent macroscale. Particular attention has been given to internal stagnation point flows, such as cross-slot flows [11,12]. Pathak and Hudson [11] used flow-induced birefringence and micro-particle image velocimetry to examine the flow of wormlike micellar fluids in a microfluidic cross-slot and observed a steady-flow asymmetry at high Weissenberg numbers. Arratia et al. [12] used particle tracking velocimetry and tracer experiments to study the flow of dilute polymer solutions through a cross-slot device and observed two types of flow instabilities that occur as the Deborah number is increased. The first instability consists of a transition to steady asymmetric flow, in which the flow remains steady but spatial symmetry is broken; the second instability, in which the flow becomes time-dependent, follows at higher Deb-

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orah numbers. A similar behavior was predicted numerically by Poole et al. [13] in an analogous geometry. They used a finite-volume method to simulate the flow of a viscoelastic fluid described by the upper-convected Maxwell model, and demonstrated that the steady asymmetry can be predicted and is purely elastic in nature. In fact, they reported that inertia has a stabilizing effect, delaying the onset of asymmetric flow and reducing the magnitude of the asymmetry. Xi and Graham [14] have also performed numerical calculations to simulate viscoelastic fluid flow through a 2D cross-slot geometry at low Reynolds numbers. In their finite-element simulations, which use a FENE-P constitutive equation, the solvent viscosity ratio is high and they only predict the existence of a time-dependent instability at high Deborah numbers. Furthermore, a recent study by Rocha et al. [15], who extended the work of Poole et al. [13] by considering models other than the UCM and exploring a wider range of model parameters (such as the solvent viscosity ratio and extensibility parameter), clearly show that as the solvent viscosity ratio is increased, the critical Deborah number becomes higher and eventually the unsteady instability is reached before the pitchfork bifurcation, the outcome observed in the study of Xi and Graham [14]. It will be shown here that an identical situation arises for the flow-focusing device.

A sound physical explanation for the mechanism generating the instability reported in the experiments of Arratia et al. [12] and the simulations of Poole et al. [13] has not been fully established yet, although Poole et al. [13] presented some evidence for compressive stresses in the two incoming streams distorting the velocity field in such a way that, coupled with streamline curvature, a destabilization similar to that occurring in curvilinear Couette flow would arise. It is not the purpose of the present study to provide a definite answer regarding the generating mechanism question, rather our main motivation for using a simple flow-focusing geometry was to explore the possibility of attaining a region of constant strain-rate by making use of opposing lateral fluid streams that shape a third inlet stream flowing perpendicularly to the lateral entrances. A similar geometry has been used by Luo [16] to explore electrokinetic instability effects to promote mixing. The extensional flow in such a device is studied in detail by Oliveira et al. [17]. While examining the effect of operating and geometric parameters on the flow, we observed the onset of purely elastic instabilities similar to those mentioned previously. A rigorous characterization of the transition from steady symmetric flow to steady asymmetric flow is the main focus of this work and the numerical results are examined in Section 4. In the previous sections we layout the characteristics of the flow-focusing geometry (Section 2) and present an overview of the numerical method and computational meshes used (Section 3). We conclude the paper with a brief summary of our findings.

2. Flow geometry and dimensionless numbers

The flow-focusing device under consideration is shaped much like a conventional "cross-slot" except that it contains three inlets and one exit channel. The geometric configuration used is illustrated in Fig. 1, where the main variables are identified. The geometry is two-dimensional and symmetric about the plane $x = 0$, with the origin of the coordinate system set at the center of the geometry.

Side streams are introduced into the central mainstream through two lateral channels of equal dimensions. The width of the lateral channels (D_2) was varied from $0.3D_1$ to $2D_1$, while the width of the outlet channel (D_3) was kept equal to D_1, the width of the central inlet channel. To account for the effects of geometric parameters, we define the relative width of the entrance branches as $WR = D_2/D_1$. For all configurations tested, the length of the inlet and outlet channels was set to 30 times the central inlet channel width ($30D_1$). Increasing the channel lengths to $60D_1$ was found to have a negligible effect on the flow patterns and critical Deborah numbers.

The average velocity ratio ($VR = U_2/U_1$), defined as the ratio of the inlet average velocities in the side streams (U_2) to the average velocity in the central inlet stream (U_1), was varied between 1 and 500.

To characterize the degree of elasticity we make use of the Deborah number, which represents the ratio between the relaxation time of the fluid (λ) and a characteristic time scale of the flow (t_{flow}), here defined as:

$$De = \lambda/t_{flow} = \lambda U_2/D_1 \tag{1}$$

For a given geometry, the Deborah number was varied between 0 (corresponding to Newtonian fluid flow) and the value corresponding to the onset of time-dependent flow. For reasons of rheological simplicity, most of the results presented here are for the upper-convected Maxwell model (UCM). We also present a limited number of simulation results using the linear form of the Phan-Thien–Tanner model (PTT), for which the extra-stress tensor is given by [18]:

$$\left[1 + \frac{\lambda\varepsilon}{\eta_p}\mathrm{tr}(\boldsymbol{\tau})\right]\boldsymbol{\tau} + \lambda\overset{\nabla}{\boldsymbol{\tau}} = 2\eta_p\mathbf{D} \tag{2}$$

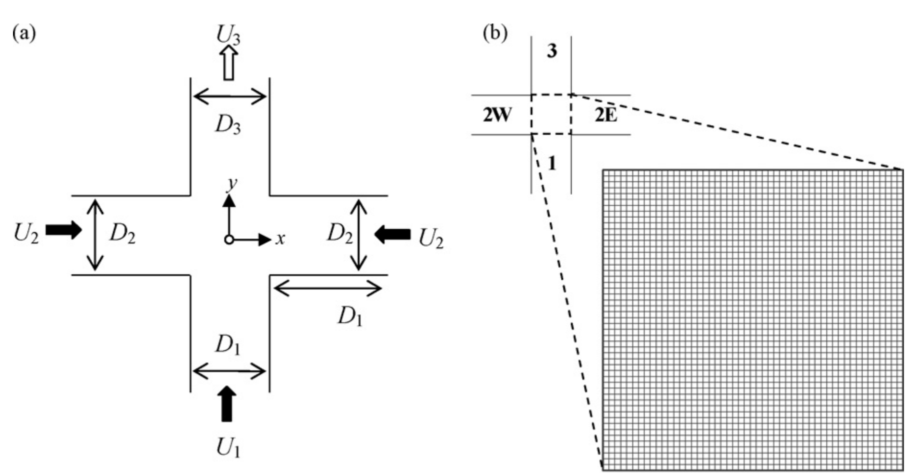

Fig. 1. (a) Schematic of the flow-focusing geometry, where U_i are mean velocities in channels of width D_i and lengths of $30D_1$; (b) details of the central region of the standard computational mesh used to map the geometry with $WR = 1$ ($\Delta x_{\min} = \Delta y_{\min} = 0.02D_1$).