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Velocity overshoots in gradual contraction flows

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ABSTRACT

This study reports the results of a systematic numerical investigation, using the upper-convected Maxwell (UCM) and Phan-Thien–Tanner (PTT) models, of viscoelastic fluid flow through three-dimensional gradual planar contractions of various contraction ratios with the aim of investigating experimental observations of extremely large near-wall velocity overshoots in similar geometries [R.J. Poole, M.P. Escudier, P.J. Oliveira, Laminar flow of a viscoelastic shear-thinning liquid through a plane sudden expansion preceded by a gradual contraction, Proc. Roy. Soc. Lond. Ser. A 461 (2005) 3827]. We are able to obtain good qualitative agreement with the experiments, even using the UCM model in creeping-flow conditions, showing that neither inertia, second normal-stress difference nor shear-thinning effects are required for the phenomenon to be observed. Guided by the numerical results we propose a simple explanation for the occurrence of the velocity overshoots and the conditions under which they arise.

1. Introduction

Experimental velocity measurements of the flow of a high-molecular weight flexible polymer solution through planar *gradual* contraction–sudden expansion geometries [1,2] have revealed an interesting fluid-dynamic effect. Spanwise[1] profiles of the streamwise velocity in the *XZ*-centreplane exhibited extreme velocity overshoots close to the sidewalls, up to three times the centreline velocity in magnitude, that due to their appearance were called "cat's ears". More recent experiments, without the sudden expansion component, have confirmed that the appearance of the "cat's ears" profiles are a sole consequence of the smooth contraction [3]. Representative velocity profiles are reproduced in Fig. 1(a), together with a schematic of the contraction geometry used in the experiments, in which a representative velocity profile along the spanwise (neutral) direction is illustrated (Fig. 1(b)).

Three-dimensional viscoelastic calculations using the Phan-Thien–Tanner (PTT) model [4] have been attempted to match the experimental conditions of Ref. [2] with a few limited simulations reported in Poole et al. [2] and an extended systematic study reported in Afonso and Pinho [5]. Although some modest success in predicting velocity overshoots was achieved, the magnitude of the overshoots – at most about 10% higher than the centreline velocity – was always much lower than that observed in the experiments. To capture these weak overshoots the full PTT model was required ($\xi \neq 0$ producing $N_2 \neq 0$ in steady simple shear flow) together with strong strain hardening (low values of ε) and some inertia. In these simulations it was speculated that the presence of the geometric singularity due to the sudden expansion prevented convergence at higher Deborah numbers and that, if convergence could be achieved, a non-zero second normal-stress difference may not be required for the effect to be observed (i.e. $\xi \neq 0$ just allowed the *De*–*Re* space to be reached where "cat's ears" occur).

Our interest in the current study is to revisit the problem in an attempt to capture the extreme nature of the "cat's ears" effect and to try to reveal the mechanism for their appearance. To do so our approach, in contrast to the simulations of Refs. [2,5], is to concentrate on modelling a gradual contraction section alone, as in the recent experiments of Keegan et al. [3]. Furthermore we set aside the goal of trying to exactly match the experimental conditions of Refs. [1,2] by selecting a related, but simplified, 3D-geometry and by varying the *Re* and *De* numbers in a systematic way. Using such a methodology we are able to show that, even for the rheologically "simple" UCM model, extreme velocity overshoots can be predicted even in the absence of inertia, i.e. the velocity overshoots are a purely *elastic* effect. Thus "cat's ears" profiles appear to be an inherent feature of viscoelastic flow through gradual contractions provided certain conditions, which we identify based on our numerical results, are satisfied.

The rest of this paper is organised as follows; in Section 2 we briefly describe the equations to be solved, the numerical method, the geometry and the meshes used; in Section 3 we discuss the

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[1] Here we use the terms streamwise for the flow (x) direction and spanwise for the neutral (z) direction.

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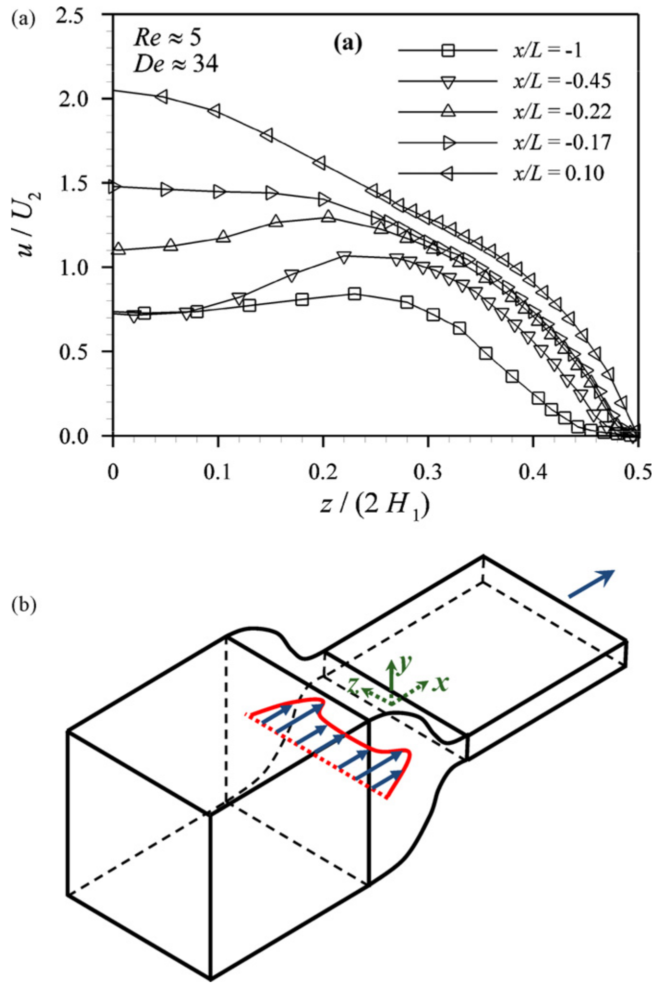

Fig. 1. (a) Spanwise profiles of the streamwise velocity for an aqueous solution of 3000 ppm polyacrylamide in an 8:1 planar gradual contraction flow $Re \approx 5$, $De \approx 34$ (adapted from Ref. [3]) and (b) schematic of the geometry and coordinate axis.

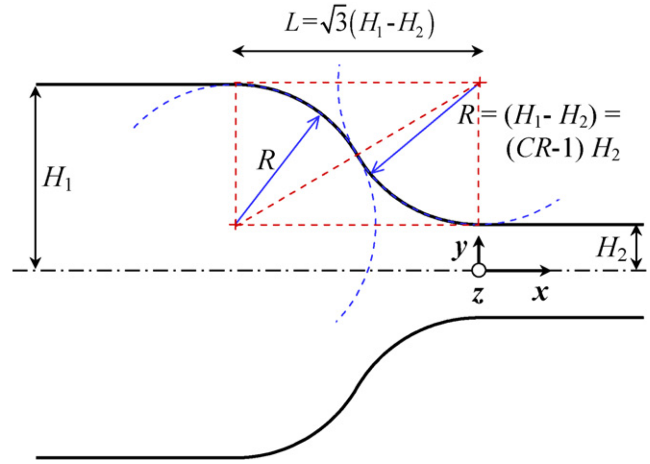

Fig. 2. Schematic of planar gradual contraction geometry. The depth of the geometry in the z-direction is constant ($2\,H_1$) (adapted from Ref. [10]).

results for a Newtonian fluid followed by the results of the viscoelastic models in Section 4; in Section 5, based on our numerical results, we discuss a possible mechanism for the "cat's ears" effect before summarising our findings in Section 6.

2. Governing equations, numerical method, geometry and computational meshes

We are concerned with the isothermal flow of an incompressible viscoelastic fluid through a gradual three-dimensional planar contraction geometry. The equations to solve are those of conservation of mass:

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

and of momentum:

$$\rho \left[\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} \right] = -\nabla p + \nabla \cdot \boldsymbol{\tau}. \tag{2}$$

For reasons of rheological simplicity most of the simulations we report here are for the well known upper-convected Maxwell (UCM) model [6]; in addition some simulations are conducted for the simplified version of the Phan-Thien and Tanner model (PTT) [4] of which the UCM is a limiting case:

$$\lambda \left[\frac{\partial \boldsymbol{\tau}}{\partial t} + \nabla \cdot \boldsymbol{u}\boldsymbol{\tau} \right] + f(\mathrm{Tr}\,\boldsymbol{\tau})\boldsymbol{\tau} = \eta_p(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}) + \lambda(\boldsymbol{\tau} \cdot \nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}} \cdot \boldsymbol{\tau}). \tag{3}$$

Eq. (3) retains only the upper-convected part of the full Gordon-Schowalter derivative. In the current study the stress function $f(\mathrm{Tr}\,\boldsymbol{\tau})$ takes the linear form proposed in Ref. [4]:

$$f(\mathrm{Tr}\,\boldsymbol{\tau}) = 1 + \frac{\lambda \varepsilon}{\eta_p} \mathrm{Tr}(\boldsymbol{\tau}). \tag{4}$$

In Eqs. (3) and (4) the constant model parameters are the relaxation time of the polymer λ, the zero-shear polymer viscosity η_p and the extensibility parameter ε. Setting the ε parameter to zero produces the UCM model. The UCM model exhibits both a constant shear viscosity η and first normal-stress coefficient (and hence relaxation time) allowing us to explore the effects of elasticity without the complications of shear-thinning of either the shear viscosity or relaxation time. In contrast the PTT model exhibits shear-thinning of both these parameters but has the benefit of bounded extensional stresses in purely extensional flow enabling higher Deborah numbers to be reached.

A fully implicit finite-volume numerical method is used to solve Eqs. (1)–(4). The original numerical method, and subsequent developments, has been described in great detail in Refs. [7–9] and so is not unnecessarily repeated here.

The gradual contraction geometries we investigate here are essentially three-dimensional versions of the geometries we used to investigate the phenomenon of "divergent flow" in Ref. [10]. A schematic 2D projection of the geometry used in the numerical simulations is shown in Fig. 2. The geometry consists of two ducts, the larger (inlet) one being square in cross-section and having a half-height H_1 and the other (entrant) having the same width but different height ($2H_2$) connected by two arcs (one convex, the other concave) of constant radius of curvature, $R = H_1 - H_2$. Defining the contraction ratio as CR ($=H_1/H_2$) we can also express this radius of curvature as $R = (CR - 1)H_2$. The coordinate system is set on the XY and XZ symmetry planes at the "entrance" to the smaller channel. Although not identical to the geometries used in Refs. [1–3], this choice enables consistency between geometries of differing contraction ratio and also a constant wall radius of curvature. Despite these small differences the essential nature of the experimental geometry used in Ref. [3] is maintained.