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A 1.5D numerical model for the start up of weakly compressible flow of a viscoplastic and thixotropic fluid in pipelines

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ABSTRACT

We investigate the problem of the start up of a compressible flow in a pipeline filled with a viscoplastic and thixotropic material. The objective of this study is to examine the possibility that the flow can restart for a pressure drop below the value predicted by the conservative relation $\Delta\hat{p} = 4\hat{\tau}_y\hat{L}/\hat{D}$, where $\hat{\tau}_y$ denotes the yield stress, $\hat{L}$ the pipe length and $\hat{D}$ the pipe diameter, thanks to the combined effects of compressibility and thixotropy. Our numerical model is a compromise between a fully 2D model and a fully 1D model, i.e., a 1.5D model. Only the velocity component in the direction of the pipe axis is assumed non-zero but it is allowed to vary both in axial and radial directions. We show that this intermediate model yields accurate results which are consistent with the predictions of the fully 2D model. The gain in computing time while keeping an equivalent reliability in the numerical predictions enables us to investigate the effects of the compressibility and thixotropy dimensionless numbers at a reasonable cost.

1. Introduction

In the last decade, the exploration of new oilfields has resulted in the production of considerable quantities of waxy crude oil (about 20% of the world petroleum reserves). Handling waxy crude oil under steady flow and reasonable temperature conditions is a relatively easy task. However, the high wax content of these oils can lead to difficulties when the temperature drops [1–3]. Most of the complexity is related to paraffin crystals forming an interlocking gel-like structure that changes the crude oil rheological properties [4]. For various reasons (maintenance, emergency, etc.), flow shutdowns may occur, which allows time for significant temperature change to occur in the pipeline. As the temperature drops below the wax apperance temperature (WAT), the first paraffin crystals appear. Slightly below the WAT, due to the build up of a crystallized paraffin network, the material starts to gel and to exhibit non-Newtonian properties comprising non-zero yield stress, shear-thinning viscosity, temperature dependence and weak compressibility [5]. This complex behavior has been evidenced in many experimental studies [6–11]: the lower the temperature, the higher the yield stress and the viscosity. The change in the rheology might be very severe: the viscosity and/or yield stress of the oil can vary over a range of several decades with a 20 °C temperature drop. The main difficulty with waxy crude oil transportation is the issue of restarting a pipeline filled with a gelled oil. The start up scenario is very basic. Under standard conditions of use, the waxy crude oil flows steadily in the pipeline. As the flow is shut down, the oil starts to cool down and its behavior changes once the temperature drops (usually slightly) below the WAT, due to the gel structure build up. As the flow is restarted in the pipe, the gel structure is destroyed as a result of shear, up to the point where it has been completely broken down and the flow has recovered to steady flow conditions.

Waxy crude oils are deemed to be thixotropic material in the sense that their rheological properties (viscosity and yield stress) depend on the mechanical and temperature history. In other words, the level of viscosity and yield stress is affected by the current temperature and shear-rate to which the material is subjected but also by the way in which it has been cooled down and its entire shear-rate history. The level of gelling, or structuring, which determines the viscosity and yield stress magnitude, is often modeled by a structure parameter. For convenience, this parameter lies in the range 0–1, where 0 implies a fully broken down structure and 1 a fully built up structure. In the most severe situations, the waxy crude oil is fully gelled in the pipeline and the structure parameter is assumed to be equal to 1. As the flow is restarted (if the inlet pressure is large enough), shear effects destroy the gel structure, which means that the structure decreases from 1 towards 0. As a result, both the yield stress and viscosity decay, the flow rate in the pipe increases and in turn the shear rate increases, which accelerates the

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structure breakdown mechanism. It is therefore a self-reinforcing mechanism.

Let us assume now that the pipe is filled with a fully gelled oil. The theoretical minimal pressure drop $\Delta\hat{p}_{\min}$ to restart the flow is given by the following relation:

$$\Delta\hat{p}_{\min} = \frac{4\hat{\tau}_y(\hat{t}=0)\hat{L}}{\hat{D}} \tag{1}$$

where $\hat{\tau}_y(\hat{t}=0)$ is the magnitude of the yield stress at the time of restart, $\hat{L}$ the pipe length and $\hat{D}$ the pipe diameter. Under incompressible conditions, if the inlet pressure is not high enough to overcome the yield stress, i.e., $\Delta\hat{p} \leq \Delta\hat{p}_{\min}$, the flow never starts and no structure breakdown mechanism occurs. If the inlet pressure is indeed large enough to overcome the yield stress, i.e., $\Delta\hat{p} > \Delta\hat{p}_{\min}$, the flow will restart, no matter whether the material has a structure or not.

Now, if the material is weakly compressible, (as waxy crude oil is, due to thermal shrinkage at low temperatures and the presence of gas voids), the above statements still hold for $\Delta\hat{p} > \Delta\hat{p}_{\min}$ but not in all cases for $\Delta\hat{p} < \Delta\hat{p}_{\min}$, since the oil is thixotropic. The restart mechanism is fairly simple to explain and works in the following manner. Let us assume that the waxy crude oil is compressible and thixotropic. Let us assume also that a pressure drop slightly below $\Delta\hat{p}_{\min}$ is applied between the pipe inlet and oulet. Since the flow is weakly compressible, the material starts to move in the pipeline (see our previous work on compressible viscoplastic flows [12]), which creates an initial shear. This shear initiates structure breakdown. As a result, both yield stress and viscosity drop and eventually, at the end of the compressible phase $\hat{t} = \hat{t}_{cp}$, if the yield stress $\hat{\tau}_y(\hat{t} = \hat{t}_{cp})$ is low enough such that the following relation:

$$\Delta\hat{p} > \int_0^{\hat{L}} \frac{4\hat{\tau}_y(\hat{t}=\hat{t}_{cp})}{\hat{D}}\mathrm{d}z$$

is fulfilled, the flow restart is successful. This fairly simple mechanism prompts the following statement: there exist situations where, although the pressure drop is below the theoretical minimal pressure drop, the flow restarts thanks to the combined effects of thixotropy and compressibility. One of the objectives of this paper is to validate and quantify this statement. At the practical level, this should convince pipeline designers that they usually over-estimate the minimal pressure required to restart pipelines filled with gelled waxy crude oil.

To investigate the issue of restarting a compressible flow of a viscoplastic and thixotropic material in a pipeline, we propose to use a new 1.5D model. In this model, there is only one non-zero velocity component, in the axial direction of the pipeline, but this component depends both on the axial and radial position. Thus the name “1.5D”, meaning a compromise between a 1D and a 2D model. In our previous works [5,12], we proposed a fully 2D model to examine the waxy crude oil flow in pipelines and focused on the temperature effects in steady flowing conditions and compressible ones in restart situations. Governing equations were discretized by a finite volume scheme and the solution algorithm relies on Lagrange multiplier technique and Uzawa/augmented Lagrangian method. This 2D model has proved to provide reliable and promising results but at the cost of time consuming computations (in particular due to the relatively slow, though highly robust, convergence properties of the augmented Lagrangian solution algorithm). For the aforementioned reason, we developed in our next papers simpler 1D models. In [13] we focused on single-phase compressible restart flows and in [14] we incorporated the displacement of the outgoing gelled fluid by the incoming fresh oil injected at the pipe inlet, via a lubrication modelling approach. The 1D model is highly tractable and has produced valuable results. However, the 1D nature prevents a detailed description of the structure breakdown mechanism. Structure breakdown is driven by shear, which varies significantly along the pipe radius, as in any standard Poiseuille flow. Assuming the structure to be constant per cross-section is a very poor assumption. It is highly unlikely that the use of an averaged shear rate in the pipe cross-section, as in a fully 1D model, would provide the right structure breakdown rate, which is crucial to determine whether the flow restarts or not (recall that the restart capability relies on how fast the structure breakdown occurs in relation to the compressible transient). This is a serious drawback of our 1D model that we aim to correct in our new 1.5D model where the radial variation of the velocity, and thus of the shear rate, is accounted for.

The main “ingredients” of our 1.5D model are a blend of the 1D and 2D models. We start by writing the governing equations in the framework of the lubrication assumption but stop one step before averaging across the pipe cross-section, as in [13]. The resulting system of equations is discretized using a finite volume scheme and the solution is provided by an Uzawa/augmented Lagrangian algorithm as for the 2D model. As an outcome, we end up with a tractable model which is almost as accurate as the fully 2D one but which allows efficient computations in the sense that the computing time is closer to the one of the fully 1D model (usually, the 1.5D model computing time is a few (around 10) times larger than the one of the 1D model, whereas the 2D model computing time is at least 100 times larger).

The rest of the paper is organized as follows. In Section 2 we detail the approach used to simplify the governing equations, where we keep only the leading order terms. The solution algorithm is shortly reviewed in Section 3 together with the finite volume scheme adapted to the special 1.5D form of the governing equations. Computed results presented in Section 4 illustrate the ability of the model to provide results similar to the fully 2D model and to investigate, at a more reasonable cost, the combined effects of thixotropy and compressibility on the restart of the flow. Finally, we give concluding remarks and perspectives of this work in Section 5.

2. Description of the problem

We start by presenting the governing equations in a general multi-dimensional framework. Then we redefine the practical problem in terms of dimensionless groups. As a final step, we simplify the system of equations in light of the lubrication assumption. Throughout the whole paper, we shall work with dimensionless quantities, and distinguish any required dimensional quantities with a “hat” symbol, $\hat{\cdot}$.

2.1. Governing equations

Let Ω be a bounded domain of $\mathbb{R}^d$ and $[0, \hat{T}]$ a time interval. The transient, isothermal and compressible flow of a viscoplastic and thixotropic fluid is governed by the following equations:

- *Continuity equation:*

$$\frac{D\hat{\rho}}{D\hat{t}} + \hat{\rho}\nabla\cdot\hat{\boldsymbol{u}} = 0 \quad \text{in } \Omega\times[0,\hat{T}] \tag{2}$$

where $\hat{\rho}$ is the density, $(D/D\hat{t})$ the convective time derivative defined as $(D/D\hat{t}) = (\partial/\partial\hat{t}) + \hat{\boldsymbol{u}}.\nabla$ and $\hat{\boldsymbol{u}}$ the velocity vector with $\hat{\boldsymbol{u}} = (\hat{u}, \hat{v}, \hat{w})$. ∇ and $\nabla\cdot$ denote the gradient and divergence operators respectively.

Compressibility is introduced through the pressure dependence of the density $\hat{\rho}(\hat{p})$. Using the *isothermal compressibility* $\hat{\mathcal{X}}_\Theta = (1/\hat{\rho})\left(\partial\hat{\rho}/\partial\hat{p}\right)_\Theta$, measuring the compressibility due to the