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Shear banding and interfacial instability in planar Poiseuille flow

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ABSTRACT

Motivated by the need for a theoretical study in a planar geometry that can easily be implemented experimentally, we study the pressure driven Poiseuille flow of a shear banding fluid. After discussing the "basic states" predicted by a one-dimensional calculation that assumes a flat interface between the bands, we proceed to demonstrate such an interface to be unstable with respect to the growth of undulations along it. We give results for the growth rate and wavevector of the most unstable mode that grows initially, as well as for the ultimate flow patterns to which the instability leads. We discuss the relevance of our predictions to the present state of the experimental literature concerning interfacial instabilities of shear banded flows, in both conventional rheometers and microfluidic channels.

1. Introduction

Complex fluids have internal mesoscopic structure that is readily reorganised by an imposed shear flow. This reorganisation in turn feeds back on the flow field, resulting in strongly nonlinear constitutive properties. In some systems this nonlinearity is so pronounced that the underlying constitutive curve relating shear stress T_{xy} to shear rate $\dot{\gamma}$ in homogeneous flow is predicted to have a region of negative slope $\mathrm{d}T_{xy}/\mathrm{d}\dot{\gamma} < 0$ [1,2]. In this regime, an initially homogeneous flow is unstable to the formation of coexisting shear bands of differing local viscosities and internal structuring, with band normals in the flow-gradient direction y [3]. The signature of this transition in bulk rheometry is the presence of characteristic kinks, plateaus and non-monotonicities in the composite flow curve [4]. Explicit observation of the bands is made using local rheological techniques such as flow birefringence [5] and nuclear magnetic resonance [6,7], ultrasound [8,9], heterodyne dynamic light scattering [10,9] or particle image [11] velocimetry. Using these methods, the existence of shear banding has been firmly established in a wide range of complex fluids, including wormlike [4–6,12–19] and lamellar [20–25] surfactants; side-chain liquid crystalline polymers [26]; viral suspensions [27,28]; telechelic polymers [29]; soft glasses [30–32]; polymer solutions [33]; and colloidal suspensions [34].

Beyond the basic observation of shear banding, experiments with enhanced spatial and temporal resolution have more recently revealed the presence of complex spatio-temporal patterns and dynamics in many shear banded flows [8,24,25,33,35–49]. In many such cases, the bulk stress response of the system to a steady imposed shear rate (or vice versa) is intrinsically unsteady, showing either temporal oscillations or erratic fluctuations about the average (flow curve) value. Local rheological measurements reveal such signals commonly to be associated with a complicated behaviour of the interface between the bands [8,24,33,35,36,39,41–43,47–49]. The majority of these measurements have been in one spatial dimension (1D), normal to the interface between the bands. However 2D observations in Refs. [47,48] explicitly revealed the presence of undulations along the interface, in a boundary driven curved Couette flow, accompanied by Taylor-like vortices [49]. The undulations were shown to be either static or dynamic, according to the imposed flow parameters.

Theoretically, instability of an initially flat interface between shear bands was predicted in boundary driven planar Couette flow in Refs. [50–52]. In this work, separate 2D studies in the flow/flow-gradient (x–y) [50,51] and flow-gradient/vorticity (y–z) [52] planes revealed instability with respect to undulations along the interface with wavevector in the flow and vorticity directions respectively. In both cases the mechanism for instability was suggested to be a jump in normal stress across the interface [53].

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While these predictions provide a good starting point, there remains the possibility that the interfacial undulations observed in Refs. [47–49] originate instead in curvature driven effects such as a bulk viscoelastic instability of the Taylor Couette [54] type in the high shear band, as discussed in Ref. [49]. These were neglected in the planar calculations of Refs. [50–52] (Other possibilities, also neglected, include free surface instabilities at the open rheometer edges; and an erratic stick-slip motion at the solid walls of the flow cell. We shall not consider these further in what follows here either.)

In principle, therefore, either (or both) of (at least) two possible mechanisms could underlie the observed interfacial undulations: (i) a bulk viscoelastic Taylor Couette like instability of the strongly sheared band, or (ii) instability of the interface between the bands, driven by the normal stress jump across it. Of these, scenario (i) can only arise in a curved geometry. A possible experimental route to discriminating between these two scenarios is to perform rheology on the pure high shear branch, thereby eliminating the interface required for (ii). This is technically difficult, although the results of Ref. [55] suggest stability of the high shear phase alone. Another possible way to compare (i) and (ii) is to perform experimental studies in a planar flow geometry, thereby eliminating the curvature required for (i). However they are technically difficult to implement in a boundary driven setup.

There thus exists a clear need for theoretical predictions in a planar flow geometry that could easily be implemented experimentally. An obvious candidate comprises pressure driven flow in a rectilinear microchannel of rectangular cross-section with a high aspect ratio $L_z/L_y \gg 1$. Indeed, such experiments have recently been performed [56–58], as discussed in more detail below. With this motivation in mind, in this paper we study the planar Poiseuille flow of a shear banding fluid driven along the main flow direction x by a constant pressure drop $\partial_x P = -G$. For simplicity we assume the fluid to be sandwiched between stationary infinite parallel plates at $y = \{0, L_y\}$, neglecting the lateral walls in the z direction, and so taking the limit $L_z/L_y \to \infty$ at the outset. Our main contribution will be to show an interface between shear bands to be unstable in this pressure driven geometry, as it is in the boundary driven planar Couette flow studied previously [50–52]. We will furthermore give results for the growth rate and wavevector associated with the early stage kinetics of this instability, as well as the ultimate flow patterns to which it leads.

The paper is structured as follows. After introducing the rheological model and boundary conditions in Section 2, we calculate in Section 3 the one-dimensional (1D) shear banded states that are predicted when spatial variations are permitted only in the flow-gradient direction y, artificially assuming translational invariance in x and z, and accordingly assuming a flat interface between the bands. These form the "basic states" and initial conditions to be used in the stability calculations of the rest of the paper.

In Section 4 we study the linear stability of these 1D basic states with respect to small amplitude perturbations with wavevector $q_x\hat{\mathbf{x}}$ in the flow direction. As in the case of boundary driven flow studied previously, we find an undulatory instability of the interface between the bands [50–52]. Results are then presented for the ultimate nonlinear dynamical attractor in this x–y plane, from simulations that adopt periodic boundaries in x. This exhibits interfacial undulations of finite amplitude that convect along the flow direction at a constant speed. In Section 5 we turn instead to the flow-gradient/vorticity plane y–z, likewise demonstrating linear instability of the interface with respect to small amplitude perturbations with wavevector $q_z\hat{\mathbf{z}}$. We also give results for the ultimate nonlinear flow state, which in this plane is steady. Directions for future work, which will include full 3D calculations, are discussed in Section 6.

2. Model and geometry

The generalised Navier–Stokes equation for a viscoelastic material in a Newtonian solvent of viscosity η and density ρ is

$$\rho(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = \nabla\cdot(\mathbf{T} - P\mathbf{I}) = \nabla\cdot(\mathbf{\Sigma} + 2\eta\mathbf{D} - P\mathbf{I}), \quad (1)$$

where $\mathbf{v}(\mathbf{r}, t)$ is the velocity field and $\mathbf{D}$ is the symmetric part of the velocity gradient tensor, $(\nabla\mathbf{v})_{\alpha\beta} \equiv \partial_\alpha v_\beta$. Throughout we will assume zero Reynolds' number $\rho = 0$; and fluid incompressibility,

$$\nabla\cdot\mathbf{v} = 0. \quad (2)$$

The quantity $\mathbf{\Sigma}(\mathbf{r}, t)$ in Eq. (1) is the extra stress contributed to the total stress $\mathbf{T}(\mathbf{r}, t)$ by the viscoelastic component. We assume this to obey the diffusive Johnson–Segalman (DJS) model [59,60]

$$(\partial_t + \mathbf{v}\cdot\nabla)\,\mathbf{\Sigma} = a(\mathbf{D}\cdot\mathbf{\Sigma} + \mathbf{\Sigma}\cdot\mathbf{D}) + (\mathbf{\Sigma}\cdot\mathbf{\Omega} + \mathbf{\Omega}\cdot\mathbf{\Sigma}) + 2G_0\mathbf{D} - \frac{\mathbf{\Sigma}}{\tau} + \frac{\ell^2}{\tau}\nabla^2\mathbf{\Sigma}. \quad (3)$$

Here a is a slip parameter describing non-affinity of molecular deformation, i.e., the fractional stretch of the polymeric material with respect to that of the flow field. For $|a| < 1$ (slip) the intrinsic constitutive curve is capable of the non-monotonicity of Fig. 1, thereby admitting a shear banding instability. G_0 is a plateau modulus, τ is the viscoelastic relaxation time, and $\mathbf{\Omega}$ is the antisymmetric part of the velocity gradient tensor. The diffusive term $\nabla^2\mathbf{\Sigma}$ is needed to correctly capture the structure of the interface between the shear bands, with a slightly diffusive interfacial thickness $O(l)$, and to ensure unique selection of the shear stress at which banding occurs [61].

Within this model we study flow between infinite flat parallel plates at $y = \{0, L_y\}$. The fluid is driven in the positive x direction by a constant pressure gradient $\partial_x p = -G$, the plates being held stationary. Accordingly, we write Eq. (1) (at the zero Reynolds number of interest here) in the form

$$0 = \nabla\cdot(\mathbf{\Sigma} + 2\eta\mathbf{D} - \tilde{P}\mathbf{I}) + G\hat{\mathbf{x}}, \quad (4)$$

in which we have separated the main driving pressure gradient from the rest of the pressure field, $\tilde{P}$. Fluid incompressibility is enforcing by casting the fluid velocity in terms of streamfunctions; and $\tilde{P}$ is eliminated by taking the curl of Eq. (4). (The $q_x = 0$, $q_z = 0$ part of Eq. (4), containing G but not $\tilde{P}$, is dealt with

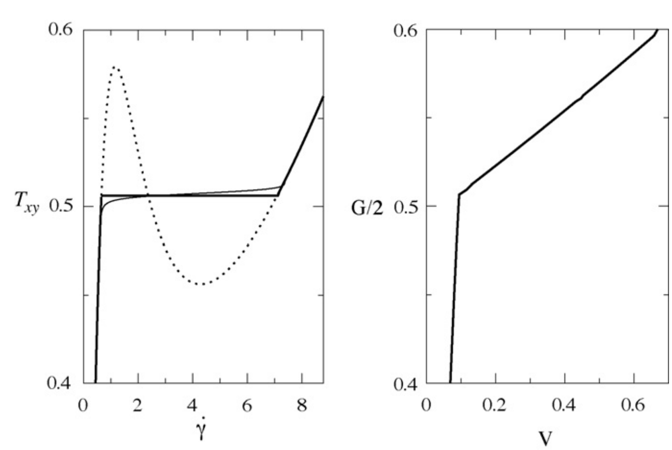

Fig. 1. *Left*: Dotted line: homogeneous constitutive curve for $a = 0.3$, $\eta = 0.05$. Thick solid line: composite flow curve for one-dimensional planar shear banded Couette flow (data already published in Ref. [50]). Selected stress $T_{sel} = 0.506$. Thin solid line: parametric plot of local shear stress $T_{xy}(y) = \Sigma_{xy}(y) + \eta\dot{\gamma}(y) = G((1/2) - y)$ against local shear rate $\dot{\gamma}(y)$ for one-dimensional planar shear banded Poiseuille flow with $l = 0.00125$, $G = 2.0$. *Right*: Halved pressure gradient versus total throughput for one-dimensional planar Poiseuille flow with $a = 0.3$, $\eta = 0.05$, $l = 0.0025$. This shows a kink at the onset of banding at $G/2 = T_{sel}$ as expected.