J. Non-Newtonian Fluid Mech. 144 (2007) 49–57

Journal of Non-Newtonian Fluid Mechanics

www.elsevier.com/locate/jnnfm

Heat transfer from a neutrally buoyant sphere in a second-order fluid

G. Subramanian [a], Donald L. Koch [b,*]

[a] *Engineering Mechanics Unit, Jawaharlal Nehru Centre for Advanced Scientific Research, Jakkur, Bangalore 560064, India*

[b] *School of Chemical Engineering, Cornell University, Ithaca, NY 14853, USA*

Received 31 August 2005; received in revised form 15 February 2007; accepted 21 February 2007

Abstract

In the absence of inertial effects, the heat or mass transfer from torque-free neutrally buoyant spheres in planar shearing flows of Newtonian fluids is diffusion limited at large *Pe*; here, the Peclet number (*Pe*) is a dimensionless measure of the relative importance of convective and diffusive transfer mechanisms. In the inertialess Newtonian limit, the linearity and reversibility of the governing Stokes equations of motion leads to the existence of a region of closed streamlines around the freely-rotating particle that precludes convective enhancement in these flows. Non-Newtonian stresses act to break this symmetry, and the rate of heat/mass transfer from the particle is significantly increased for large *Pe*. It is possible to analytically determine the transport rate in the limit of weak non-Newtonian effects. For a torque-free particle in a planar linear flow of a second-order fluid, the dimensionless rate of heat or mass transfer, characterized by the Nusselt number, is found to be $Nu = 0.478[Pe\,De(1+\lambda)^2(1+\epsilon)]^{1/3}$ in the limit $Pe\,De \gg 1$ and $De \ll 1$, where ϵ is a dimensionless property of the fluid, and λ is a parameter that depends on the relative magnitudes of extension and vorticity in the ambient flow. In simple shear flow, corresponding to $\lambda = 0$, the Nusselt number may be alternatively written as $6.01(Pe\,De)^{1/3}[((1/2)+(\Psi_2/\Psi_1))/(1+(\Psi_2/\Psi_1))]^{1/3}$; here, Ψ_1 and Ψ_2 are the first and second normal stress coefficients. The Deborah number (*De*) is the ratio of the intrinsic relaxation time of the fluid to the macroscopic flow time scale in all above cases, and serves as a dimensionless measure of the relative magnitudes of the non-Newtonian (elastic) and Newtonian stresses; for simple shear flow, one may define $De = (\Psi_1 + \Psi_2)\dot{\gamma}/\eta$ in terms of the normal stress coefficients, η being the solvent viscosity.
© 2007 Elsevier B.V. All rights reserved.

Keywords: Second-order fluid; Linear flow; Heat transfer; Closed streamlines; Symmetry; Convective enhancement; Boundary layer

1. Introduction

The transport of heat/mass from neutrally buoyant particles suspended in shearing flows of viscoelastic fluids is relevant to several industrial applications; for instance, in the sterilization of food products. Heat and mass transfer to suspended particles also play a role in certain polymerization processes such as emulsion and suspension polymerization. Although the particles involved in these applications may not have mass densities that are precisely matched to those of the suspending fluids, the high viscosity of most viscoelastic fluids implies that the effects of sedimentation may be weak. From a fundamental point of view, it is of interest to examine how elastic forces act to change the heat/mass transfer characteristics of particles suspended in non-Newtonian fluids when compared to the Newtonian scenario. In addition, the changes in the fluid velocity field on account of elastic effects, and the resulting paths of fluid elements, are expected to yield useful insights into the more difficult problem of interaction between two or more spheres. This is in light of the fact that inertialess pair-interactions in a Newtonian fluid subject to a linear flow, notwithstanding differences in detail, closely follow the topology of the streamlines around a single torque-free sphere [1,2]. An understanding of pair-interactions in a non-Newtonian fluid may help to explain the initial stages of particle aggregation in sheared viscoelastic suspensions. The latter phenomenon often complicates the interpretation of rheological measurements [3]; experiments have revealed the tendency of spheres in shearing flows to form long chains along the flow direction [4]. Clearly, characterizing the flow field around a single torque-free sphere in the shearing flow of a second-order fluid represents a first step towards understanding pair-interactions in viscoelastic media.

In the Newtonian case, the transport phenomena in the absence of inertia remain diffusion limited even at large *Pe*. Here, *Pe* is a dimensionless parameter that governs the relative importance of the convective and diffusive transport mechanisms; thus,

* Corresponding author. Tel.: +1 607 255 3484; fax: +1 607 255 9166.
E-mail address: dlk15@cornell.edu (D.L. Koch).

0377-0257/$ – see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.jnnfm.2007.02.013

for particles of radius a suspended in a flow with a characteristic velocity gradient of $\dot{\gamma}$, the relevant velocity scale is $O(\dot{\gamma}a)$, and $Pe = \dot{\gamma}a^2/\alpha$, α being the thermal diffusivity.[1] The absence of the familiar convective enhancement is owing to the existence of closed streamlines in a thin annulus surrounding the freely-rotating sphere in a planar linear flow, as a result of which fluid elements sufficiently close to the sphere circulate around it in periodic orbits with axes in the vorticity direction. Convection is therefore no longer effective in carrying heat away from the particle. One does not observe the familiar boundary layer enhancement of heat transfer wherein the dimensionless heat transfer, characterized by the Nusselt number (Nu), grows as an algebraic power of Pe [5]. In contrast, the Nusselt number for a torque-free particle in a planar shearing flow tends to an $O(1)$ constant for $Pe \to \infty$ [6]. Here, the Nusselt number is defined as $Q/(4\pi ka\Delta T)$, Q being the dimensional rate of heat transfer, k the coefficient of thermal conductivity and ΔT the temperature difference between the particle surface and the ambient fluid, so that $Nu = 1$ in the conduction limit. In the inertialess limit, the motion of the Newtonian fluid is governed by the quasi-steady Stokes equations, and it is the linearity and reversibility of these equations [7] that leads to closed streamlines in a reference frame that translates with the neutrally buoyant particle.

One expects that a generic perturbation to the aforementioned degenerate situation will induce a topological change in the streamline pattern, in turn significantly altering the rate of heat transfer from the suspended particle. Recently, Subramanian and Koch [8,9] have shown that fluid inertia breaks the closed streamline topology. The resulting spiralling flow has a bi-axial extensional character, the compressional axis being coincident with the ambient vorticity direction, and convects heat away from the sphere in an efficient manner. In this paper, the physical mechanism that allows for a similarly efficient transfer at large Pe is the non-Newtonian rheology of the suspending fluid. In order to make analytical progress, we consider the limit of weak elastic effects. In other words, the Deborah number, $De = \dot{\gamma}\tau$, is assumed to be much smaller than unity, so the suspending fluid's rheology is governed by the retarded motion expansion [10–12], and to $O(De)$, satisfies the second-order fluid constitutive equations; here, τ is a characteristic relaxation time of the fluid. Note that cases of oscillatory (or transient) shear flows allow one to define two distinct dimensionless measures of non-Newtonian effects—the ratios of the characteristic relaxation time τ to the shear rate, and to the time period of oscillation (ω^{-1}). These are known, sometimes interchangeably, as the Deborah and the Weissenberg numbers (We). However, for the steady flows considered here, there exist only two time scales, and both De and We contain the same physics; we have used the former to denote the ratio of these two time scales. Also, note that the high viscosities of most viscoelastic fluids ensure that inertial forces play a negligible role, and we will assume this to be the case.

[1] The present findings are, of course, equally applicable to both heat and mass transfer and α may be replaced by the mass diffusivity D in the latter case. However, for ease of description, we restrict ourselves to the former from hereon.

In a manner similar to the action of inertial forces, the non-linearity of the equations of motion of a second-order fluid breaks the symmetry associated with the Stokes limit. For an ambient two-dimensional linear flow with a non-dimensional velocity gradient tensor given by

$$\mathbf{\Gamma} = \begin{bmatrix} \frac{1+\lambda}{2} & \frac{1-\lambda}{2} & 0 \\ -\frac{1-\lambda}{2} & -\frac{1+\lambda}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

λ being a measure of the relative magnitudes of extension and vorticity in the ambient flow, the resulting spiralling motion converges along the vorticity axis and spreads outward along the equatorial plane of the rotating sphere, thereby resembling the inertial flow induced by centrifugal forces [9]. Throughout this paper, all positions will be non-dimensionalized by the sphere radius a, velocity gradients by $\dot{\gamma}$, and stresses by $\mu\dot{\gamma}$, where μ is the zero shear rate viscosity of the suspending fluid. In the limit $Pe \gg 1$, a standard boundary layer analysis applied to the azimuthally averaged $O(De)$ flow yields an analytical expression for the Nusselt number; the azimuthal coordinate is defined in a plane perpendicular to the ambient vorticity axis. In particular, we find that $Nu = 0.478(Pe\,De)^{1/3}[((1/2) + (\Psi_2/\Psi_1))/(1 + (\Psi_2/\Psi_1))]^{1/3}$ in simple shear flow, where Ψ_1 and Ψ_2 are the first and second normal stress coefficients defined in the usual manner [10].[2]

The paper is organized as follows. In Section 2 we write down the expression for the velocity disturbance, to $O(De)$, due to a neutrally buoyant torque-free sphere in the linear flow of a second-order fluid. Thereafter, we plot sample trajectories obtained from a numerical integration of this velocity field to illustrate the breaking of the Newtonian symmetry in simple shear flow, that then enables convection to carry heat away from the sphere. We also schematically depict the changes in the three-dimensional streamline topology at $O(De)$ for this case. Later, in Section 3, we employ a perturbation expansion, valid for small De, to solve the convection-diffusion equation for the temperature field. In a spherical polar coordinate system aligned with the vorticity direction of a planar linear flow, it is shown that one only need consider the $O(De)$ convection in the radial and meridional directions for large Pe; close to its surface, the leading order convection along the azimuthal coordinate does not carry heat away from the sphere. The equation for the azimuthally averaged temperature field is then solved in the limit of large Pe using a boundary layer analysis. The thickness of the thermal boundary layer is only $a(PeDe)^{-(1/3)}$ in the limit $Pe\,De \gg 1$, and it is therefore sufficient to retain the approximate forms of the radial and meridional velocity components near the surface of the sphere. Using these we obtain an analytical expression for the Nusselt number as a function of Pe, De and λ. Finally, in Section 4, we conclude with a discussion of the main results.

[2] Note that, in simple shear flow, $De = (\Psi_1 + \Psi_2)\dot{\gamma}/\eta$, so the expression for the Nusselt number is, in fact, independent of $(\Psi_1 + \Psi_2)$, and the singularity at $\Psi_2/\Psi_1 = -1$ is only an apparent one.

Download English Version:

https://daneshyari.com/en/article/671352

Download Persian Version:

https://daneshyari.com/article/671352

Daneshyari.com