ELSEVIER

J. Non-Newtonian Fluid Mech. 148 (2008) 141–150

Journal of Non-Newtonian Fluid Mechanics

www.elsevier.com/locate/jnnfm

Steady flow across a confined square cylinder: Effects of power-law index and blockage ratio

A.K. Dhiman [a], R.P. Chhabra [a,*], V. Eswaran [b]

[a] Department of Chemical Engineering, Indian Institute of Technology, Kanpur 208016, India
[b] Department of Mechanical Engineering, Indian Institute of Technology, Kanpur 208016, India

Abstract

The effect of blockage ratio on the flow characteristics of power-law fluids across a square cylinder confined in a channel has been investigated for the range of conditions $1 \le Re \le 45$, $0.5 \le n \le 2.0$ and $\beta = 1/8$, 1/6 and 1/4. Extensive numerical results on the individual and total drag coefficients, wake length, stream function, vorticity and power-law viscosity on the surface of the square cylinder are reported to determine the combined effects of the flow behavior index, blockage ratio and Reynolds number. The size of the wake region is influenced more by blockage than by power-law index. Similarly, drag is also seen to be more influenced by blockage ratio and the Reynolds number than that by the power-law index.

Keywords: Non-Newtonian fluid; Steady flow; Blockage ratio; Square cylinder; Drag

1. Introduction

In recent years, considerable interest has been shown in studying the flow of Newtonian fluids past cylinders of circular and square cross-sections oriented normal to the direction of flow. From a theoretical stand point, the flow past a square cylinder has received attention due to the variety of flow phenomena (various flow regimes) exhibited under appropriate kinematic conditions. Naturally, these changes also manifest at the macroscopic level in the way the drag coefficient, wake length depend on the Reynolds number, and on whether the flow is confined or unconfined. Furthermore, a reliable knowledge of engineering parameters (drag coefficient, wake size, etc.) is frequently needed for the design of cooling towers, support structures, etc. Consequently, over the years, a wealth of information on flow and heat transfer characteristics has accumulated in the literature, for circular cylinders [1–4] and square cylinders [5–15]. Since excellent and extensive reviews of the pertinent studies are available elsewhere [16–20], only the salient features are recapitulated here. Most of the currently available literature on the incompressible fluid flow over a confined square cylinder relates to the high Reynolds number region where the main thrust has been to investigate the wake phenomena, time-dependent drag and lift characteristics, vortex shedding frequency, etc. Recently, Dhiman et al. have studied the effects of Reynolds ($1 \le Re \le 45$) and Prandtl numbers ($0.7 \le Pr \le 4000$) on the flow of Newtonian fluids and heat transfer across a square cylinder for both unconfined [16] and confined [17,18] configurations in cross-flow. Subsequently, this study has been extended to the flow and/or heat transfer of power-law fluids past an unconfined square cylinder [19–21] in the steady flow regime.

Much less is known about the effect of blockage on the flow of and heat transfer to power-law fluids, even for circular cylinders [22,23], although D'Alessio and Pascal [24], Chhabra et al. [25] and Soares et al. [26] have reported the effect of the domain to gradually diminish as the Reynolds number is increased for the cross-flow of power-law fluids over a circular cylinder. This finding is qualitatively consistent with the experimental observations made in the free fall conditions [27,28]. Broadly, an increase in the blockage ratio and/or a decrease in the value of the power-law index has qualitatively similar effects on the streamline and vorticity patterns.

For the analogous 2D, steady flow past a square cylinder, as far as known to us, there has been only one numerical study [29] for a single value of the blockage ratio of 1/8. Using a rather coarse and uniform grid over the range of conditions as $5 \le Re \le 40$, $5 \le Pe \le 400$ and $0.5 \le n \le 1.4$. It was found that shear-thinning behavior not only reduces the size of the recirculation region, but also delays the onset of wake formation, and shear-thickening fluids show the opposite effect on the wake

* Corresponding author. Tel.: +91 512 2597393; fax: +91 512 2590104.
E-mail address: chhabra@iitk.ac.in (R.P. Chhabra).

0377-0257/$ – see front matter
doi:10.1016/j.jnnfm.2007.04.010

formation. Also, the temperature field was seen to decay very quickly at high Peclet numbers in shear-thinning fluids, with the reverse behavior being observed in shear-thickening fluids.

Overall, shear-thinning fluid behavior facilitates heat transfer, whereas shear-thickening behavior impedes it. Subsequently, Nitin and Chhabra [30] extended this work for the power-law flow past an obstacle of rectangular cross-section for the identical ranges of physical parameters. Therefore, only preliminary and scant numerical results are available in the literature on the effects of blockage ratio for the flow of power-law fluids past a square cylinder even in the steady cross-flow regime ($1 \le Re \le 45$). Thus, this study aims to explore the effect of power-law index (n) on the flow across an isolated square cylinder confined by planar walls for the range of conditions $1 \le Re \le 45$ and $0.5 \le n \le 2.0$ and for three values of the blockage ratio, $\beta = 1/8$, 1/6 and 1/4 which are well within the 2D steady flow regime.

2. Problem statement and formulation

The system of interest here is the steady bounded 2D flow of an incompressible power-law fluid in a channel with a square cylinder placed symmetrically on the centerline, as shown in Fig. 1. The square cylinder with side b, also the non-dimensionalizing length scale, is exposed to a fully developed (parabolic for a Newtonian fluid) velocity field with a maximum velocity of $U_{\max}$ at the channel inlet. The non-dimensional upstream distance between the inlet plane and the front surface of the cylinder is X_u/b and the downstream distance between the rear surface of the cylinder and the exit plane is X_d/b with the total length of the computational domain being L_1/b in the axial direction. The non-dimensional vertical distance between the upper and lower bounding walls, L_2/b in the lateral direction, defines the blockage ratio ($\beta = b/L_2$).

The dimensionless forms of the continuity, the x- and y-components of Cauchy's equations, are given below. Since the governing field equations are given in detail elsewhere [19], these are presented only briefly here.

The power-law fluid behavior is represented by

$$\tau_{IJ} = 2\eta\varepsilon_{IJ} \tag{1}$$

where τ_{IJ} and ε_{IJ} are the components of the stress and of the rate of deformation tensors, respectively, e.g., $\varepsilon_{xx} = \partial u/\partial x$, $\varepsilon_{yy} = \partial v/\partial y$, $\varepsilon_{xy} = 1/2[(\partial u/\partial y) + (\partial v/\partial x)]$ and the power-law viscosity is given by,

$$\eta = \left(\frac{\prod_\varepsilon}{2}\right)^{(n-1)/2} \tag{2}$$

where $\prod_\varepsilon$ is the second invariant of the rate of deformation tensor, which for this flow in the cartesian coordinates is related to the derivatives of the velocity components as:

$$\prod\nolimits_\varepsilon = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \tag{3}$$

Combining Eqs. (1)–(3) with the momentum equations, the governing equations in their conservative form are written as follows

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

x-Momentum:

$$\frac{\partial u}{\partial t} + \frac{\partial (uu)}{\partial x} + \frac{\partial (vu)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\eta}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \frac{2}{Re}\left(\varepsilon_{xx}\frac{\partial \eta}{\partial x} + \varepsilon_{yx}\frac{\partial \eta}{\partial y}\right) \tag{5}$$

y-Momentum:

$$\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{\partial (vv)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\eta}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{2}{Re}\left(\varepsilon_{yy}\frac{\partial \eta}{\partial y} + \varepsilon_{xy}\frac{\partial \eta}{\partial x}\right) \tag{6}$$

where Re is the Reynolds number defined as $\rho U_{\max}^{2-n} b^n / m$. Here, m and ρ are the power-law consistency index and the density of the fluid, respectively.

The corresponding dimensionless boundary conditions for this flow configuration may be written as follows (Fig. 1):

- At the inlet boundary:

  $u = 1 - (2\beta y)^{n+1/n}; \quad v = 0$

  where $\beta = b/L_2$ and $0 \le y \le L_2/2b$
- At the upper and lower boundary walls:

  $u = 0; \quad v = 0$ (no-slip condition)
- At the surface of the square cylinder:

  $u = 0; \quad v = 0$ (no-slip condition)
- At the exit boundary:

  The homogeneous Neumann boundary condition, i.e., $\partial\phi/\partial x = 0$ is employed here, where ϕ is the dependent variable, u or v.

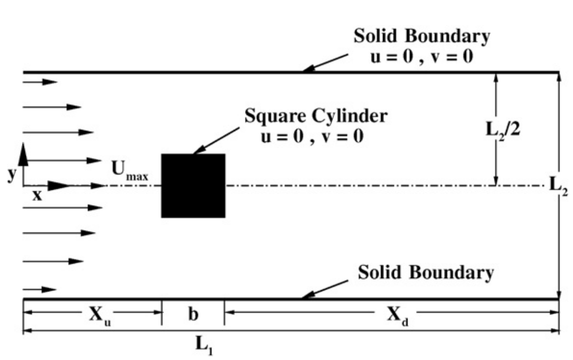

Fig. 1. Schematics of the flow around a square cylinder confined in a channel.