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Very slow flow of Bingham viscoplastic fluid around a circular cylinder

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ABSTRACT

Numerical simulations have been used to study the flow of a Bingham viscoplastic fluid around a circular cylinder in an infinite medium with negligible inertia effects. Papanastasiou's regularisation technique has been adopted to approximate the model. The case corresponding to preponderant plasticity effects has been particularly studied and convergence of the solutions examined in detail. The flow kinematics and stresses have been determined. The rigid zones have been identified and characterised. At large Oldroyd numbers, when plasticity effects become preponderant, a viscoplastic boundary layer appears around the cylinder. The characteristics of this viscoplastic boundary layer are quantified. The results are compared with existing theoretical results, concerning particularly the predictions of the viscoplastic boundary layer theory and the plasticity theory.

1. Introduction

The presence of viscoplastic fluids (fresh concrete, sludge, mud flows, food pastes, cosmetic fluids, etc.) in very many areas of life has generated increased interest in their study.

In spite of the difficulties inherent in studying such fluids, significant results have been obtained in recent years. With regard to the basic fluid mechanics problem of viscoplastic fluid flow around obstacles, emphasis has been placed more especially on spheres, for which interesting studies have been performed [1–8].

As far as circular cylinders are concerned, one of the first studies (a semi-analytical study) was carried out by Adachi and Yoshioka [9]. Other authors have also looked at the case of circular cylinders via a numerical approach [10–13]. It is to be noted that most of the numerical studies carried out on the flow of a viscoplastic fluid around a circular cylinder have concerned only the asymptotic case in which plasticity effects are negligible (i.e. where the fluid behaves almost as a Newtonian fluid) or relatively low-plasticity effects. To our knowledge, the second asymptotic case of preponderant plasticity effects for which the fluid becomes plastic and rigid, has only been considered by Mitsoulis [11], who has examined the wall influence on the rigid zones. This author also provided drag coefficients but only for weak and intermediate plasticity effects. The kinematic field was not dealt with in this study.

The aim of this present work is therefore to examine in detail cases corresponding to very high-Oldroyd numbers Od (defined as the ratio of yield stress effects to viscous effects) by extending the study to kinematics and dynamics aspects, by performing finite-element numerical simulations. The emphasis will be placed on the effects of numerical parameters on flow characteristics. Wall effects will be investigated and the condition of an infinite medium will be looked for. Drag coefficients will be calculated. Velocity and stress fields, as also changes in shape, size and position of the rigid zones will be presented in a large range of Oldroyd numbers ($Od \in [10; 2 \times 10^5]$).

Because of the lack of theoretical results concerning plasticity effects in this type of flow, the link between viscoplastic fluid flows and perfectly plastic flows will be considered. Then, the results obtained at very high-Oldroyd numbers will be compared with the predictions of plasticity theory [1,14–16]. The other results obtained at relatively low-Oldroyd numbers will be compared with existing results [9–13].

Oldroyd [17] developed the viscoplastic boundary layer theory, in which he showed that at very high-Oldroyd numbers the material becomes rigid everywhere except in a thin layer, where viscous and plastic effects are both important, that he named “viscoplastic boundary layer”. Piau [18], Piau and Debiane [19] re-examined and corrected Oldroyd's theory. These authors studied the case of very slow flow of a Bingham viscoplastic fluid around a flat plate. The characteristics of this flow, in particular the change in thickness of the viscoplastic boundary layer along the plate and the drag force, were expressed analytically as a function of the Oldroyd number. The numerical results of this paper will be examined in light of this theory.

2. Problem formulation

The problem under consideration concerns the very slow flow of a Bingham viscoplastic fluid around a circular cylinder of diameter d

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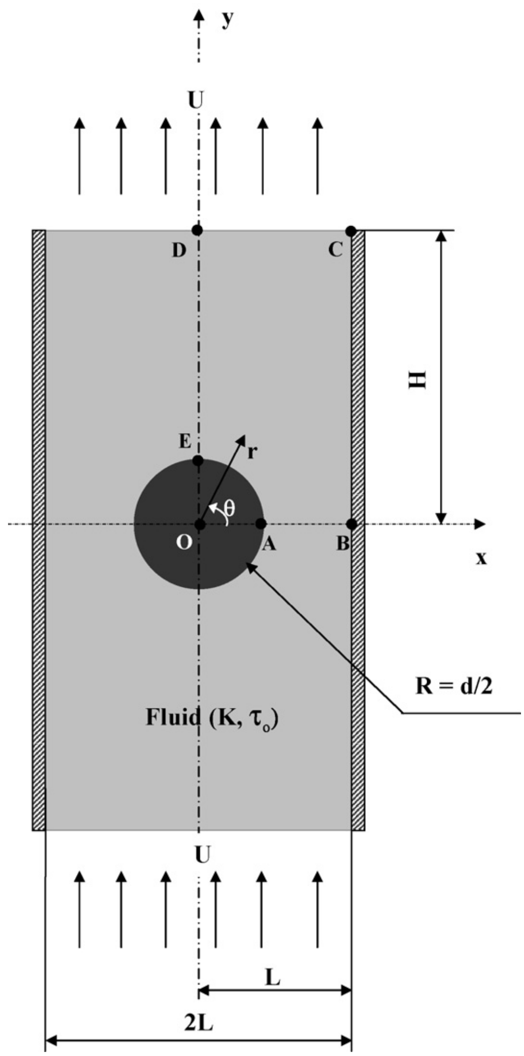

Fig. 1. Flow field. Schematic representation of the problem.

situated half-way between two flat plates 2*L* apart (Fig. 1) in such a way that the medium may be considered as infinite. The flow length is 2*H*. The velocity of the fluid far from the cylinder is constant and denoted by *U*.

The fluid is incompressible and the flow, assumed to be steady and isothermal, is governed by the mass and momentum conservation equations, which, in the absence of inertia, are written as follows:

$$\nabla \cdot \vec{V} = 0, \tag{1}$$

$$\nabla \cdot \underline{\underline{\tau}} - \nabla p = 0, \tag{2}$$

$$\underline{\underline{\sigma}} = -p\underline{\underline{I}} + \underline{\underline{\tau}}, \tag{3}$$

where $\vec{V}$ is the velocity vector, p the pressure, $\underline{\underline{\sigma}}$ the Cauchy stress tensor and $\underline{\underline{\tau}}$ is its deviatoric part.

The Bingham constitutive equation based on the von Mises criterion [20,21] is given by

$$\begin{cases} \underline{\underline{\tau}} = 2\left(K + \dfrac{\tau_0}{2\dot{\gamma}}\right)\underline{\underline{\dot{\gamma}}}, & \text{if } \tau > \tau_0 \\ \underline{\underline{\dot{\gamma}}} = 0, & \text{if } \tau \le \tau_0 \end{cases}, \tag{4}$$

where τ_0 and K are respectively the yield stress and fluid consistency coefficient, $\underline{\underline{\dot{\gamma}}}$ the strain rate tensor. $\dot{\gamma} = \sqrt{(1/2)\,\mathrm{tr}\left(\underline{\underline{\dot{\gamma}}}^2\right)}$ and $\tau = \sqrt{(1/2)\,\mathrm{tr}\left(\underline{\underline{\tau}}^2\right)}$ are the second invariant of the strain rate tensor and the second invariant of the stress deviator tensor, respectively.

The only dimensionless number of the problem is the Oldroyd number (Od), defined as

$$Od = \frac{\tau_0 d}{KU}. \tag{5}$$

All the lengths are non-dimensionalised by the cylinder diameter *d*, and the velocities by the velocity *U* of the fluid far from the cylinder. Pressures and stresses are non-dimensionalised by the yield stress τ_0 in relation to our interest in very slow flow.

As inertia is neglected, two planes of symmetry may be considered. The domain of study is thus reduced to the quarter OBCD of the flow field (Fig. 1) and the prescribed boundary conditions are expressed as follows:

- on the vertical plane of symmetry (ED), Vx = 0 and $\partial Vy/\partial x = 0$;
- on the horizontal plane of symmetry (AB), Vx = 0 and $\partial Vy/\partial y = 0$;
- according to the no slip assumption on the surface of the cylinder, Vx = 0 and Vy = 0;
- on the boundaries (BC) and (CD), Vx = 0 and Vy = *U*;
- pressure is zero-referenced at point B.

The drag coefficient will be presented according the two following formulae:

$$C_\mathrm{d} = \frac{F}{A(KU/d)} = \frac{F}{lKU} \text{ (viscous drag coefficient)}, \tag{6}$$

$$C_\mathrm{d}^* = \frac{F}{A\tau_0} = \frac{C_d}{Od} \text{(plastic drag coefficient)}, \tag{7}$$

where *F* denotes the drag force exerted by the fluid on the cylinder and *A* = *ld*, the front area of the cylinder obtained on a plane perpendicular to the flow direction. *l* is the length of the cylinder.

3. Numerical method

3.1. Numerical model

It is very difficult to use the original form of viscoplastic fluid constitutive equations for numerical studies owing to their discontinuity. The method most commonly applied to overcome this difficulty is to regularise these equations. Several authors [22,23] have examined the most commonly used regularisation models (simple model, bi-viscous model and Papanastasiou's model) for various types of flow and have compared them with the exact solutions. It appears, at least for relatively low-plasticity effects and for these types of flow, that Papanastasiou's model [24] with a suitable regularised parameter provides the best velocity of convergence and accurately predicts shape and position of the rigid zones.

Regularisation of the Bingham constitutive equation with Papanastasiou's model gives the following equation:

$$\underline{\underline{\tau}} = 2\left\{K + \frac{\tau_0(1 - \mathrm{e}^{-m\dot{\gamma}})}{2\dot{\gamma}}\right\}\underline{\underline{\dot{\gamma}}}, \tag{8}$$

where *m* denotes the regularisation parameter.

The problem posed by using the regularisation method lies in the fact that the rigid zones in which the strain rate should be zero are approximated by regions with very low-strain rates. In this study, a special care will be paid to the choice of the parameters of regularisation.

The calculations have been carried out with the finite-element software Polyflow (Fluent Inc.). The problem variables are the pressure and the two velocity components. We have adopted quadrilateral finite elements with a quadratic approach for the