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J. Non-Newtonian Fluid Mech. 138 (2006) 161–172

Journal of Non-Newtonian Fluid Mechanics

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Analysis of the flow of a power-law fluid film on an unsteady stretching surface by means of homotopy analysis method

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Received 11 April 2005; received in revised form 1 May 2006; accepted 5 May 2006

Abstract

Flow of a power-law fluid film on an unsteady stretching surface is analyzed by means of homotopy analysis method (HAM [1]). For real power-law index and the unsteadiness parameter in wide ranges, analytic series solutions are given and compared with the numerical results. The good agreement between them shows the effectiveness of HAM to this problem. Additionally, unlike previous studies, the value of the critical unsteadiness parameter S_0, above which no solution exists, is determined analytically in this paper.

Keywords: Homotopy analysis method; Power-law fluid film; Unsteady stretching surface; Analytic series solution; Non-Newtonian fluid

1. Introduction

The problem of flow within a thin liquid film due to the stretching of a surface is often encountered in manufacturing processes. For example, during mechanical forming processes, such as extrusion, melting–spinning, the extruded material issues through a die. The ambient fluid condition is stagnant but a flow is induced close to the material being extruded, due to the moving surface. Similar situations prevail during the manufacture of plastic and rubber sheets where it is often necessary to blow a gaseous medium through the not-yet-solidified material, and where the stretching force may be varying with time. All coating processes demand a smooth glossy surface to meet the requirements for best appearance and optimum service properties such as low friction, transparency and strength (see Andersson et al. [2]). Wang [3] was the first to consider the flow problem within a finite liquid film of Newtonian fluid over an unsteady stretching sheet. Later on, Usha and Sridharan [4] considered a similar problem of axisymmetric flow in a liquid film. Recently, Andersson et al. [5] had explored the heat transfer characteristics of the hydrodynamic problem solved by Wang [3]. The influence of thermocapillarity on the flow and heat transfer in a thin liquid film was investigated by Dandapat et al. [6].

Noting that the fluids employed in material processing or protective coatings are in general non-Newtonian, the study of non-Newtonian liquid films are important. The free-surface flow of non-Newtonian liquids in thin films is a widely occurring phenomenon in various industrial applications, for instance in polymer and plastic fabrication, food processing and in coating equipment. Andersson and Irgens [7] have presented a comprehensive review article of the relevant papers on film flow of power-law fluids. Although there are a few papers on gravity-driven power-law film flows [8–10], the studies of non-Newtonian film flows on an unsteady stretching surface remain little. Andersson et al. [2] carried out a numerical analysis for the hydrodynamic problem of power-law fluid flow within a liquid film over a stretching sheet. The heat transfer aspect of such problem has been considered recently by Chen [11].

In this paper, we examine analytically the flow of an incompressible non-Newtonian liquid film over an unsteady stretching surface, which had been considered by Andersson et al. [2], however, using a kind of numerical method. The method we employed

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doi:10.1016/j.jnnfm.2006.05.011

here is based on the homotopy analysis method (HAM [1]), a newly developed tool for non-linear problems. In order to implement this method, a new similarity transformation is introduced to transform the extent of the independent variable into a finite range of 0–1. For real power-law index and the unsteadiness parameter in wide ranges, analytic series solutions are given and compared with numerical results. We hope that the analytic solutions given by HAM is helpful to understand the behavior of such flows and might find applications in manufacturing engineering, such as polymer extrusion. Additionally, unlike previous studies, the similarity transformation employed in this paper allows the value of the critical unsteadiness parameter S_0, above which no solution exists, to be determined analytically.

2. Mathematical formulation

Consider the flow of a thin, power-law fluid film of thickness $h(x, t)$ on a horizontal elastic surface, as shown in Fig. 1. The fluid motion within the film arises due to the stretching of the elastic surface. The velocity field in the film is governed by the two-dimensional boundary layer equations (see Andersson et al. [2]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial y}\right), \tag{2}$$

where (x, y) are the Cartesian coordinates along and normal to the surface, respectively, u and v are the velocity components along the x and y directions, respectively, t is the time, ρ is the density of the fluid, and τ_{xy} represents the shear stress. In the present problem, we have $\partial u/\partial y \leq 0$, which gives the shear stress as

$$\tau_{xy} = -K\left(-\frac{\partial u}{\partial y}\right)^n, \tag{3}$$

where K is called the consistency coefficient and n is the power-law index. It should be mentioned that the so-called Ostwald-de-Waele power-law fluid model is employed here. Very good comments on this model can be found in [7]. Combining (2) and (3) we have

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\left(\frac{K}{\rho}\right)\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n. \tag{4}$$

For the particular parameter value $n = 1$, one can retrieve the Newtonian fluid model with dynamic coefficient of viscosity K. As n deviates from unity, deviations from Newtonian behavior occur. For example, $n < 1$ and $n > 1$ correspond to pseudo-plastic or shear thinning fluids and dilatant fluids or shear thickening fluids, respectively. Pseudo-plastic fluids have viscosity functions that decrease with increasing shear rate. The fluid becomes progressively less viscous and appears to become “thinner” with increasing shear rates. Dilatant fluid, on the other hand, have viscosity functions that increase with the shear rate. The fluid becomes progressively more viscous and appears to become “thicker” with increasing shear rates.

The flow is caused by the stretching of the elastic surface at $y = 0$ with a velocity

$$U = \frac{cx}{1-\alpha t} \tag{5}$$

where c and α are positive constants both with dimensions time^{-1}. Note that the analysis is valid only for time $t < 1/\alpha$.

We introduce the following similarity transformations:

$$u = cx(1-\alpha t)^{-1} f'(\eta), \tag{6}$$

$$v = -\left(\frac{c^{1-2n}}{K/\rho}\right)^{-1/(n+1)}\left[\frac{2n}{n+1}f(\eta) + \frac{1-n}{1+n}\eta f'(\eta)\right] x^{(n-1)/(n+1)}(1-\alpha t)^{(1-2n)/(1+n)}\beta, \tag{7}$$

where the similarity variable η is given by

$$\eta = \left(\frac{c^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{(1-n)/(1+n)}(1-\alpha t)^{(n-2)/(1+n)}\beta^{-1} y, \tag{8}$$

and β is a yet unknown constant denoting the dimensionless film thickness, defined by

$$\beta = \left(\frac{c^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{(1-n)/(1+n)}(1-\alpha t)^{(n-2)/(1+n)} h(x, t). \tag{9}$$