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Effect of contraction ratio upon viscoelastic flow in contractions: The axisymmetric case

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Abstract

A comprehensive numerical study of the effects of the contraction ratio upon viscoelastic flow through axisymmetric contractions was carried out. Six contraction ratios were examined (CR = 2, 4, 10, 20, 40 and 100) using the Oldroyd-B and Phan–Thien–Tanner (PTT) constitutive equations, under creeping-flow conditions and for a wide range of Deborah numbers (*De*). The results enabled the construction of vortex pattern maps, with CR and *De* as independent parameters, elucidating the role of these dimensionless groups in controlling vortex growth, vortex type (lip or corner vortices), and pressure-drop characteristics. The extensional parameter of the PTT model was also varied ($\varepsilon = 0$–0.5) and it was found that for small values of ε the Couette correction is a monotonic decreasing function of *De*, while for high ε values it is a monotonic increasing function.

Keywords: Axisymmetric contraction; Viscoelastic fluid; Creeping flow; Contraction ratio; PTT model; Oldroyd-B model

1. Introduction

The entry-flow is a long-standing problem dating back to the late 1800s [1]. The early works were mainly experimental and were primarily stimulated by the need to construct a capillary rheometer capable of measuring accurately the viscosity of Newtonian fluids [2]. Since then contraction flows have been the subject matter of numerous experimental and numerical works. The first numerical study of this problem, in which the complete equations of motion were solved, was published by Vrentas et al. [3] and concerned the creeping flow of a Newtonian fluid through an axisymmetric contraction.

In spite of the simple geometry, these flows exhibit complex patterns where shear and extensional regions co-exist. Near the walls the flow is shear-dominated while along the centerline it is purely extensional. These flows are amongst the most studied extensionally dominated flows, since they assume particular importance in industrial applications involving viscoelastic non-Newtonian fluids. Examples worth mentioning are polymer processing applications, such as injection molding, spinning and film blowing [4]. Literature on contraction flows up until the late 1980s have been the subject of detailed reviews by Boger [1] and White et al. [5], in which the complex effects of flow geometry and fluid rheology were addressed. For a comprehensive account of more recent results the reader is referred to the thorough introductions of Alves et al. [6] and Rodd et al. [7]. An overview of the evolution of numerical methods applied to the flow of viscoelastic fluids through contractions can be found in Walters and Webster [8], which includes typical constitutive models used to represent real fluids; for a more complete review on the theme refer to the book by Owens and Phillips [9]. Here we limit ourselves to a description of some important results with direct relevance to the problem at hand.

1.1. The sudden contraction configurations

There are three main types of configuration used in sudden contraction research: the axisymmetric, the planar and the three-dimensional contraction of which the square–square contraction is a particular case. In the first, the fluid flows from a capillary of *large* diameter through a sudden contraction into a

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smaller capillary; in the corresponding planar case, the capillaries are replaced by channels of large aspect ratio (quasi-2D) and the contraction occurs only in one direction, while in the square–square case this happens in two perpendicular directions.

The flow of viscoelastic fluids through sudden contractions, either planar, axisymmetric or square–square, generates complex flow patterns. In general, these flows comprise regions of strong shearing close to the walls and non-homogeneous extension along the centerline upstream and downstream of the contraction [10]. Yet, it has been shown that the fluid behavior associated with entry flows in different geometries can be quite contrasting as a result of geometric and rheological dissimilarities.

Perhaps the most widely studied configuration is the axisymmetric contraction, mainly due to its implications in pipe/duct flow [11]. Most of the early papers are devoted to the experimental study of this flow (e.g. [12–16]; more recent experimental studies can be found in [10,17–20]). Up to the early 1990s, numerical methods were usually unable to accurately predict the steady contraction flow of viscoelastic fluids, and most efforts were thus dedicated to improving the numerical techniques, as documented in the books of Crochet et al. [21] and Owens and Phillips [9].

Studies on the planar geometry were carried out soon after the pioneering investigations on axisymmetric contractions. The planar geometry assumed relevance as optical experimental techniques evolved, such as flow visualization and especially birefringence, and the number of published works on this configuration increased substantially [22–29]. The flow through planar contractions has also received additional attention in numerical investigations [30–39] since this configuration was chosen as a test case for the assessment and improvement of numerical methods in computational rheology [40].

Some of the findings observed for the axisymmetric geometries are replicated in the planar case, such as the formation of a recirculating region upstream of the contraction and the existence of an extra pressure drop associated with the flow of viscoelastic fluids. However, some major differences have been identified as a result of the different strain and strain-rate histories experienced by fluid elements in the two geometries [1]. Walters and Webster [41] found no significant vortex activity for Boger fluids in the 4:1 planar case, in marked contrast to observations in 4.4:1 circular contractions. However, for shear-thinning fluids, vortex growth was observed in both planar and axisymmetric geometries. Evans and Walters [25,26] studied the flows of shear-thinning and constant-viscosity elastic fluids and found strong vortex enhancement at all times for the shear-thinning fluids, but once more no significant vortex activity was observed for Boger fluids in planar contractions. Rothstein and McKinley [10] found that in the axisymmetric case, the size of the salient corner vortex formed was smaller than in the planar case. They attributed this outcome to the different Hencky strains experienced by the polymer molecules as the flow changes from uniaxial to planar.

Over the years, little attention has been paid to the flow through square–square contractions, most likely because they are significantly more complex, and in numerical terms a “simple” 2D simulation is clearly inadequate. In many ways, the square–square geometry can be thought of as an intermediate between the axisymmetric and the planar contraction. Geometrically, similarities between the planar and the square–square contractions are easy to picture. On the other hand, in terms of the actual flow and variation of strain-rates, similarities between circular and square–square contractions have been observed in the experimental study of Walters and Rawlinson [42]. In their study, the differences between the flow of Boger fluids in planar and axisymmetric geometries were also seen to occur between planar and square–square contractions. More recently, a few studies have addressed this geometry, both under an experimental [6] and a numerical [43] perspective.

1.2. Flow patterns

One of the remarkable flow features of viscoelastic fluids worth emphasizing is the vortex formation and vortex enhancement mechanism upstream of the contraction plane. In general, for axisymmetric contraction flows, strong vortex enhancement is observed both for Boger fluids and for shear-thinning viscoelastic fluids. However, the vortex characteristics and the way vortex enhancement evolves with the Deborah number can be strikingly different, depending on the flow geometry and the fluid rheology. In some cases the vortex forms near the salient corner and increases in strength, growing steadily upstream and radially inward towards the re-entrant corner [1,18]. In other cases the salient corner vortex grows in size with the Deborah number, while simultaneously a lip vortex forms near the re-entrant corner; subsequently, the lip vortex grows radially outwards and forces the corner vortex to decrease in both size and intensity until it is completely overtaken by the lip vortex [10].

Understanding the mechanisms underlying the flow transitions (and inherent vortex enhancement evolution) that take place as the Deborah number is varied has been a main driving force behind the experimental work in this area [9]. This evolution was shown to depend greatly on the fluid rheology. Boger et al. [16] and Boger and Binnington [18] investigated the behavior of two different Boger fluids having similar shear properties and yet found quite different vortex dynamics. It was then recognized that extensional properties had to be taken into account, and Boger [1] suggested the primary parameter to be the extensional viscosity. Recent experimental investigations by Rothstein and McKinley [10,20] corroborate the important role of the extensional viscosity on the dynamics of vortex growth and associated enhanced pressure drop. In this context one of the aims of the current paper is to examine under which conditions one pathway of vortex evolution is preferred over the other.

1.3. Excess pressure drop

In addition to the kinematics, another essential flow characteristic is the pressure drop resulting from a sudden change in diameter. The total pressure drop is a result of the pressure drop due to the fully developed viscous flow through the tubes plus the excess pressure drop associated with contraction entrance effects. Apart from being crucial for a proper assessment of the pumping power required in ducts [44,45], it yields information