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Steady extrusion of viscoelastic materials from an annular die

George Karapetsas, John Tsamopoulos *

Laboratory of Computational Fluid Dynamics, Department of Chemical Engineering, University of Patras, Patras 26500, Greece

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ABSTRACT

The steady extrusion of viscoelastic materials from a straight, annular die is studied theoretically. The viscoelastic behavior is modelled using the affine Phan-Thien and Tanner (PTT) constitutive equation of the exponential form. For the numerical solution of the governing equations the mixed finite element method is combined with a quasi-elliptic mesh generation scheme in order to capture the large deformations of the two free surfaces of the extrudate. The elastic-viscous stress splitting technique (EVSS-G) is used to separate the elastic and viscous contributions to the polymeric part of the stress tensor together with a streamline upwind Petrov-Galerkin (SUPG) weighting for the discretization of the constitutive equation. This combination of solution methods and constitutive model allows us (i) to compute accurate steady-state solutions up to very high Weissenberg numbers resulting in very high deformations of the free surfaces (ii) construct and store the Jacobian matrix, which is necessary to conduct linear stability analysis for this flow. First, results for the fully developed flow of a PTT liquid inside an annular die are presented. They reveal a complex interplay between material elasticity, shear thinning and solvent viscosity. Next, a complete parametric analysis of annular extrusion is performed. Such a complete study using the PTT model has not been reported before, even at much lower *Wi* numbers. It is found that swelling of the material increases sharply up to moderate Weissenberg numbers, whereas its rate of increase is reduced for higher values of *Wi*, as shear thinning becomes increasingly important. The latter generally plays a crucial role, in addition to elasticity, on the swelling of the extrudate. Moreover, as the contribution of the solvent viscosity increases, the contribution of elastic stresses decreases causing a decrease in the swelling of the material which approaches the Newtonian limit. The predicted swelling ratios, which characterize the geometry of the extrudate, are in satisfactory agreement with earlier experimental and theoretical data for three particular HDPE resins.

1. Introduction

The steady extrusion of a viscoelastic liquid through an annular die is an important engineering process, since it has many applications in the polymer processing industry including production of pipes, of annular preforms to be used in blow molding and of annular films to be used in film blowing and wire coating. As it is widely known, when a viscoelastic fluid is extruded through a slit, a cylindrical or an annular die, it experiences significant swelling. This is caused mainly by the relaxation of the polymeric chains, which, from being oriented primarily in the flow direction inside the die, can relax to any configuration outside it, where the flow field is completely rearranged. Since in most applications accurate dimensions of the extruded products are required, the level of swelling that the fluid experiences is an important design parameter. Therefore the annular extrusion process has been the subject of several publications in the past. Next we will mention briefly only those that seem to be most relevant to the present study.

A nice review on the experimental efforts that have been carried out by various groups to study the annular extrusion has been given by Garcia-Rejon et al. [1]. Pearson and Trottnow [2] were the first to attempt to address this problem theoretically following the ideas of Tanner [3] for the capillary die swell. A theoretical study of this problem has been reported by Crochet and Keunings [4]. They were the first to simulate the extrusion of a non-Newtonian fluid using the finite element method to solve the flow of a Maxwell fluid exhibiting, however, only small elastic effects. More recently, Luo and Mitsoulis [5] presented streamline finite element (SFEM) calculations, using the K-BKZ integral model, for a viscoelastic fluid flowing through straight, converging and diverging annular dies. Garcia-Rejon et al. [1] used the same viscoelastic model also to study the effect of the die geometry on the annular swell, although they employed the finite element method combined with the inconsistent SU4 × 4 method proposed by Marchal and Crochet [6]. The annular flow of a power-law and a second order fluid was studied by Seo [7], while shortly after Ahn and Ryan [8]

* Corresponding author. Fax: +30 2610 996 178.
E-mail address: tsamo@chemeng.upatras.gr (J. Tsamopoulos).

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used a finite difference scheme to study the flow of a power-law fluid. Tanoue et al. [9] performed viscoelastic simulations using the Giesekus model reaching high values of the Weissenberg number, although in order to achieve that, they also employed the inconsistent SU4 × 4 method. Subsequently, Tanoue and Iemoto [10] used the same method to study the effect of different annular die geometries on the flow of a Giesekus fluid. The SFEM method was also used to perform simulations for the HDPE using various viscoelastic models by Otsuki et al. [11,12]. Very recently, Mitsoulis [13] presented simulations using the regularized Herschel-Bulkley or the power-law fluid models to study the extrusion of pseudoplastic or viscoplastic fluids. The effect of surface tension and gravity acting in the flow direction on the steady annular extrusion, which could become important at small flow rates but was neglected by most researchers until then, was studied by Housiadas et al. [14] for a Newtonian fluid. The same two forces were retained also by Housiadas and Tsamopoulos [15,16], who used the thin film approximation to study the transient extrusion of annular films obeying the Oldroyd-B fluid model. More recently, Georgiou [17] presented simulations to study the effect of inertia at high Reynolds numbers on the shape of the extrudate for a Newtonian fluid.

Two are the main objectives of this paper (i) to develop an accurate and efficient numerical method in order to solve the axisymmetric and steady extrusion of a viscoelastic material following a widely accepted differential constitutive model from an annular die and perform an extensive parametric analysis to study the effect of model parameters on the shape of the extrudate and (ii) to do so in a way that the Jacobian matrix of the entire problem is generated which would be necessary in order to perform a linear stability analysis of this problem, an unresolved problem until now. The hyperbolic nature of the constitutive equations requires the values of the elastic stresses far enough inside the die as initial conditions for the two-dimensional calculations. This necessitates calculating the elastic stresses for the fully developed annular flow. To our surprise, results for this kind of flow of an exponential PTT fluid model with or without a solvent viscosity have not been reported before. In the two-dimensional numerical simulations, we use the finite element method combined with an elliptic grid generation scheme for the calculation of the unknown positions of the inner and outer free surfaces; see Dimakopoulos and Tsamopoulos [18]. We have applied this method to a number of free or moving boundary problems, such as the transient squeeze of a viscoplastic material between parallel disks, Karapetsas and Tsamopoulos [19]; the displacement of a Newtonian, a viscoelastic or a viscoplastic fluid from a tube, Dimakopoulos and Tsamopoulos [20–22]; the deformation and displacement of several bubbles inside a filament undergoing stretching, Foteinopoulou et al. [23] and the steady bubble rise and entrapment in Bingham fluids, Tsamopoulos et al. [24].

The rest of the paper is organized as follows. In Section 2, we present the governing equations and the boundary conditions for this problem. The numerical algorithm, used in our calculations, is briefly described in Section 3. In Section 4, we present the results of the extensive parametric analysis that we performed for a viscoelastic fluid undergoing either fully developed flow inside the annular die or flow inside the die and extrusion from it. Finally concluding remarks are made in Section 5.

2. Problem formulation

We consider the steady, axisymmetric flow of a viscoelastic fluid as it is extruded from an annular die. The fluid is incompressible with constant density ρ, relaxation time λ and total dynamic viscosity $\mu = \mu_s + \mu_p$, where μ_s and μ_p are the viscosities of the solvent and the polymer, respectively. The flow geometry, which is examined here, is depicted in Fig. 1. The viscoelastic fluid flows inside an annular die of inner radius R_i, outer radius R_o and length L_1 and as it exits from the die it swells and rearranges its shape. The fluid motion is simulated until far from the die exit, at distance L_2, the flow becomes fully developed. There, the inner and outer radii of the material are H_1 and H_2, respectively.

We scale all lengths with the outer radius of the annular die, R_o, and velocities with the mean velocity, V, at the die entrance. In addition both the pressure and stress components are scaled with a viscous scale, $\mu V/R_o$. Thus, the dimensionless groups that arise are the Reynolds number, $Re = \rho V R_o/\mu$, the Weissenberg number, $Wi = \lambda V/R_o$, the ratio of the solvent viscosity over the total viscosity, $\beta = \mu_s/\mu$, and the geometric ratios $l_1 = L_1/R_o$, $l_2 = L_2/R_o$, $h_i = H_i/R_o$, $h_o = H_o/R_o$ and $a = R_i/R_o$.

Inserting the previously defined characteristic quantities into the radial and axial momentum balances and the mass conservation equation, we obtain:

$$Re\, \underline{v} \cdot \underline{\nabla}\underline{v} + \underline{\nabla}P - \underline{\nabla} \cdot \underline{\underline{\tau}} = 0 \tag{1}$$

$$\underline{\nabla} \cdot \underline{v} = 0 \tag{2}$$

where $\underline{\nabla}$ denotes the gradient operator for cylindrical coordinates, $\underline{v}$ and P are the axisymmetric velocity and pressure fields, respectively, and $\underline{\underline{\tau}}$ is the extra stress tensor, which is split into a purely viscous part, $2\beta\dot{\underline{\underline{\gamma}}}$ and a polymeric contribution $\underline{\underline{\tau}}_p$:

$$\underline{\underline{\tau}} = 2\beta\dot{\underline{\underline{\gamma}}} + \underline{\underline{\tau}}_p \tag{3}$$

where $\dot{\underline{\underline{\gamma}}}$ is the rate-of-strain tensor defined as $\dot{\underline{\underline{\gamma}}} = 1/2(\underline{\nabla}\underline{v} + \underline{\nabla}\underline{v}^T)$.

To complete the description, a constitutive equation that describes the rheology of the fluid is required in order to determine the polymeric part of the extra stress tensor. As such we use the following differential model that has been proposed by Phan-Thien and Tanner [25].

$$Y(\underline{\underline{\tau}}_p)\underline{\underline{\tau}}_p + Wi\overset{\diamond}{\underline{\underline{\tau}}}_p - 2(1-\beta)\dot{\underline{\underline{\gamma}}} = 0 \tag{4}$$

where the symbol $\diamond$ over the viscoelastic stress denotes the Gordon–Schowalter derivative defined as

$$\overset{\diamond}{\underline{\underline{X}}} = \frac{D\underline{\underline{X}}}{Dt} - (\underline{\nabla}\underline{v} - \xi_s\dot{\underline{\underline{\gamma}}})^T \cdot \underline{\underline{X}} - \underline{\underline{X}} \cdot (\underline{\nabla}\underline{v} - \xi_s\dot{\underline{\underline{\gamma}}}) \tag{5}$$

where $\underline{\underline{X}}$ is any second order tensor. Two forms of the PTT model are in common use, namely the linearized form [25], where the function $Y(\underline{\underline{\tau}}_p)$ is

$$Y(\underline{\underline{\tau}}_p) = 1 + \frac{\varepsilon}{1-\beta} Wi\, tr\underline{\underline{\tau}}_p \tag{6a}$$

and the exponential form [26] with

$$Y(\underline{\underline{\tau}}_p) = \exp\left[\frac{\varepsilon}{1-\beta} Wi\, tr\underline{\underline{\tau}}_p\right] \tag{6b}$$

The linearized form of this model has been used successfully by Luo and Tanner [27] in simulating extrusion of a viscoelastic material from either a planar or a cylindrical (axisymmetric) die up to relatively high Wi numbers. In our simulations we have used the exponential form of the PTT model. Both PTT models have two parameters, ξ_s and ε. The first one is related to the non-affine motion of the polymer chains with respect to the macroscopic motion of the continuum. By setting ξ_s equal to zero no such motion or slip is allowed, the Gordon–Schowalter derivative reduces to the upper convective one and the fluid model is referred to as the affine PTT model. The other parameter, ε, imposes an upper limit to the elongational viscosity, which increases as this parameter decreases, while it introduces elongational thinning. Moreover ε is related to