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A mathematical formulation to find effective bulk and shear moduli of recycled aggregate concrete

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HIGHLIGHTS

- 6-phase model is proposed to find the homogenized linear elastic properties of RAC is used to evaluate the homogenized properties.
- Self-consistent approach is used to evaluate the homogenized properties.
- Parametric studies are performed to evaluate the effect of attached mortar on RAC.
- Parametric studies are performed to evaluate the effect of new ITZ on RAC.

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ABSTRACT

A mathematical formulation incorporating a compatible 6-phase model is proposed and implemented to find the homogenized linear elastic properties of recycled aggregate concrete (RAC) in terms of its effective bulk and shear moduli. The model comprises of five concentric spherical shells to represent new matrix, new Interfacial Transition Zone (ITZ), old matrix, old ITZ and the core raw aggregate embedded in equivalent homogenous medium by following generalized self-consistent scheme. The model is built on the primary assumption that RAC is made of only recycled aggregates, which are described by spherical inclusions of constant volume fraction and constant radii ratios of the constituent phases. Cauchy equilibrium equations and compatibility conditions are made use of to frame the relation between the variables at the constituents' interface, while Eshelby's energy equivalence is applied for solution. Validation is done by following an indirect approach, according to which the characteristic constituents of RAC – the old ITZ and old matrix – are forced to take relatively negligible values and by comparing the predicted values with those of experiments corresponding to normal aggregate concrete. Further, a parametric study is carried out for quantification of the effect of attached mortar (includes old matrix and old ITZ) and new ITZ on the elastic properties of RAC. This first attempt on analytical prediction of the effective elastic properties of RAC yields enough information to make judicious decision on the means – either remove attached mortar or improve new ITZ – to arrive at a design mix of RAC.

1. Introduction

Large demand on natural resources such as river sand and blue metal for concrete production is a motivating factor to find suitable alternatives for them. For example, it is estimated that about 40% of the natural resources extracted in USA are being directed for construction purposes [1,2]. Several efforts were made during the last 3–4 decades to reduce the dependency of concrete production on natural resources [2–4]. The best option to ensure sustained availability of raw materials for construction without having to depend on natural resources is to make use of recycled construction and demolition (C&D) wastes as substitute for the natural aggregates. Data on the amount of C&D wastes generated over the past few years in several countries is available in the literature [2,4] Replacing the natural aggregates with C&D wastes would also eliminate the ancillary problem to find landfills for dumping the C&D wastes.

Mortar attached with a recycled aggregate (RA) is an important factor that has negative influence on various properties like creep, shrinkage, etc. of recycled aggregate concrete (RAC) [5–11]. This adverse effect of the attached mortar can be reduced by using

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appropriately treated RA for the production of RAC [6,12]. The level of treatment depends mainly upon the intended application of RAC. For high end applications, generally normal aggregate concrete (NAC) is preferred as the cost of treating RA to the prescribed level is the discouraging factor. The other important factor known to influence the performance of RAC is the properties of the new interfacial transition zone (ITZ) formed between the RA and the new mortar. By adding mineral admixtures like silica fume, slag, etc., the properties of ITZ can be improved for better values of properties of RAC [13,14]. It has also been reported recently that mineral admixtures like fly-ash improve the durability of RAC as well [15]. The addition of mineral admixtures, however, has a bearing on the economy of making concrete.

Owing to availability of high performance computing, it is now possible to carry out a detailed micromechanical analysis by finite element method to find the elastic properties of NAC [16]. Sophisticated instrumentation technique like nano-indentation helps to find the material data required for such analysis. Very limited literature [7,8,17] can be found on such an analysis for RAC, because of the computational complications arising due to the presence of old ITZ and old mortar in RA [15]. Therefore, the research on RAC is mainly centered around laboratory experiments [3,18,19]. In spite of the rigour and dependability of experimental work, analytical solutions based on advanced mathematical principles and engineering knowledge can be deployed to find homogenized elastic properties of RAC.

One can find number of significant research works on homogenized linear elastic properties for NAC [20–23]. The value of bulk modulus of 2-phase composite spheres model was obtained in one of the seminal works [24] in the field of homogenization of composites. The approach, however, was able to fetch only the bounds for the shear modulus. Later, Christensen and Lo [25] introduced 'Generalized Self Consistent scheme' to obtain the value of shear modulus of the model proposed in [24]. Simenov and Ahmad [22] have used the bounds proposed by Hashin and Shtrikman [24] to evaluate the effect of transition zone on the elastic behaviour of cement-based composites. Among other aspects, they deduced that the water content has strong influence on the overall elastic properties of NAC. Ramesh et al. [23] have evaluated the elastic properties of NAC by combining the extended 2-phase composite spheres model [24] with the approach described in Christensen and Lo [25]. A major limitation of their solution is due to the assumption of constant elastic moduli for ITZ throughout its thickness. Subsequently, this constraint was relaxed to obtain the macroscopic bulk modulus of concrete by modeling smooth variation for the elastic moduli of ITZ as a power-law function of radial distance from the center of the inclusion [20]. In a parallel development, the 4-phase model [23] was deployed to obtain the effective bulk modulus of concrete by taking into consideration the effect of maximum aggregate size and aggregate gradation [26].

In this paper, a mathematical formulation incorporating a 6-phase model is proposed to find out homogenized linear elastic properties of RAC by considering it as a composite. The formulation includes a model that accommodates the 6 constituents of RAC, namely, natural aggregate, old ITZ, old mortar, new ITZ, new mortar and unknown equivalent homogeneous medium. In order to arrive at a solution, RAs are assumed to be spherical in shape, which is acceptable as stiffness is a macroscopic property [23]. Elasticity theory and Eshelby's energy equivalence relation are used to obtain the effective bulk and shear moduli of the RAC. The formulation follows the generalized self-consistent scheme. For numerical validation, the effective bulk and shear moduli values are obtained for relatively negligible values of old matrix and old ITZ. The predicted values are found to correlate well with experimentally measured moduli values of NAC. Parametric curves are developed to quantify the effect of attached mortar (old matrix) and the new ITZ on the elastic properties of RAC. Such parametric curves help to strike a balance between the level of treatment required for RA and the improvement in the properties of new ITZ to achieve a given target value for composite elastic modulus. The scope of the study is limited to evaluate only the mechanical behavior of RAC in the linear elastic range after considering its bearing on the structural design. To the best of the authors' knowledge, the present study could be the first attempt to analytically predict the effective elastic properties of RAC.

2. Proposed 6-phase model

As a first step towards model development, the geometry of RAC is defined by including additional phases to the composite spheres model by Hashin and Shtrikman [24]. The gradation of aggregates in RAC has a volume filling configuration as is also required for the extension of composite spheres model. The random distribution of aggregates in RAC allows the existence of conditions such as statistical homogeneity and macroscopic homogeneity which are important for not modeling the spatial distribution of aggregates explicitly. Further, in order to simplify the analysis, necessary assumptions are introduced while developing the model. The assumptions made by Ramesh et al. [23] are taken as the basis and appended with those specific to the old matrix and old ITZ while proposing the model. The assumptions are:

i. The aggregate inclusions are assumed as spherical for convenience in the model development and solution. This assumption is justified as the elastic properties of a composite is an average macroscopic property and is less sensitive to local effects such as stress concentration.
ii. The four constituents of RAC, namely, old ITZ, old matrix, new ITZ and new matrix are modelled as concentric spherical shells surrounding the natural aggregate.
iii. All the constituent phases of the composite including the old matrix and old ITZ are isotropic and the RAC is macroscopically isotropic.
iv. The volume fraction of inclusion and the radii ratios of the different phases are assumed to be constant for each composite sphere. It means the RAC is made up of only RAs having identical constituent phases. This enables to describe the essential characteristics of the model by using only a single RA.
v. The constituent material of each phase is assumed to be continuous and homogeneous within their bounds. In other words, the constituent material is devoid of pores or any such local discontinuities.

The proposed composite spheres model for RAC incorporating the above assumptions is shown in Fig. 1. In continuation, the 6-phase model based on Christensen's generalized self-consistent scheme for the solution of effective properties of RAC is shown in Fig. 2. The 6-phase model consists of single composite sphere embedded in an infinite medium of unknown effective properties. The phase boundaries are distinguished by the characteristic radii r_1, r_2, r_3, r_4 and r_5 as shown in the Fig. 2. Due to the assumption (iv) listed earlier, the radii ratios r_1/r_5, r_2/r_5, r_3/r_5 and r_4/r_5 are constant for each composite sphere. Further, it also implies that the model caters to finding the effective properties of composite (RAC) made of only one type of inclusion (aggregate). The elastic constants, namely, bulk modulus (k), shear modulus (μ), Poisson's ratio (υ) and Lame's constant (λ) for different phases, are distinguished by subscripts (natural aggregate by 1, old ITZ by 2, old matrix by 3, new ITZ by 4, new matrix by 5 and the unknown homogeneous