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Spheres and interactions between spheres moving at very low velocities in a yield stress fluid

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Abstract

The drag force on two spheres moving at very low controlled velocity in a viscoplastic fluid was studied as a function of the distance separating them. Two configurations were studied, namely two spheres with their centre lines along or perpendicular to the flow. The influence of the surface roughness of the spheres governing fluid adherence and slip at the wall was quantified. The drag force on an isolated sphere was also measured and used as a reference. Correlations for predicting the drag coefficient and stability criterion, with respect to sedimentation, are proposed. These results show that viscoplasticity reduces the extent of the interactions in comparison with the case of a Newtonian fluid.

Keywords: Interactions; Spheres; Yield stress fluid; Drag

1. Introduction

In order to handle concentrated suspensions of large objects in gelled fluids, which are used in numerous industrial applications, it is necessary to be able to predict stability in relation to gravity effects. This involves determining and controlling the basic mechanisms of the phenomena that govern drag at very low velocities and the interactions between objects. Many industrial applications are therefore involved. Nevertheless, knowledge on this subject is limited [1]. Chhabra [2] made an exhaustive summary of all the results obtained with respect to the behaviour of an isolated sphere in a yield stress fluid. In the case of a Newtonian fluid, it is possible to solve the problem of the interactions between two spheres analytically [3]. However, there are few results for viscoplastic fluids. Liu et al. [4] focussed on the numerical solution of interactions between two rigid spheres in a Bingham fluid. Horsley et al. [5] used an experimental approach to propose a law for predicting the sedimentation velocity of pairs of vertically aligned spheres in a viscoplastic fluid.

In this study, the authors provide new information on the interactions between two spheres of the same diameter moving at very low velocity in a viscoplastic fluid, by taking up the experimental approach of Jossic and Magnin [6]. The overall drag force of two connected spheres is measured as a function of the distance between them. The drag force was measured in quasi-static conditions with a very slow controlled velocity; this removes the uncertainty with regard to stress distribution at the surface of the sphere. Yield stress effects in this domain are very much higher than viscous effects. The yield stress of the viscoplastic model fluid was carefully characterised in the low shear rate domain involved in the experiments. Interface conditions were checked in order to control wall slip. This phenomenon is characteristic of yield stress fluids [7–9]. Spheres with different surface roughness were used for this purpose.

On the basis of the experimental results obtained, it was possible to determine a correlation for predicting the plastic drag coefficient as a function of the distance between the spheres, the surface roughness and the Oldroyd number. By comparing these experimental results with those from the literature obtained in the case of a Newtonian fluid, the authors show that viscoplasticity reduces the intensity and extent of the interactions between two spheres. In addition, the drag coefficients obtained at the lowest velocities were used to extrapolate an experimental value for the stability criterion. The stability criterion is a dimensionless number which may be used to predict whether a sphere will or will not move under gravity effects when it is placed in a yield

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Nomenclature

A	frontal area (m^2)
$Cd^* = F_d/A\tau_0$	plastic drag coefficient of an isolated sphere
d	sphere diameter (m)
D	tank diameter (m)
f	frequency (Hz)
F	force (N)
g	gravity (m s^{-2})
G', G''	elastic and viscous modulus (Pa)
H	tank height (m)
K	consistency (Pa s^n)
L	distance between spheres (m)
n	flow index
$Od = \tau_0/K(U/d)^n$	Oldroyd number
$Re = \rho U^{2-n} d^n/K$	Reynolds number
t	characteristic time (s)
U	speed (m s^{-1})
$We = t_m/t_e$	Weissenberg number
X	function of n
$Y = \tau_0/gd\,\Delta\rho$	ratio of yield stress effects to gravity effects
δ	boundary layer thickness (m)
$\dot{\gamma}$	shear rate (s^{-1})
ρ	density (kg m^{-3})
τ	shear stress (Pa)
τ_0	yield stress (Pa)
$\xi = L/d$	dimensionless distance between spheres

Subscripts

b	buoyancy
d	drag
s	side by side
w	wake

stress fluid. For a given sphere, it is thus possible to estimate the minimum yield stress required to maintain it in suspension depending on whether or not there are other spheres in the vicinity.

2. Theoretical approach

Two configurations were considered in this study for the arrangement of the spheres in relation to one another. These are represented in Fig. 1. The two spheres have the same diameter d. The distance between the centres of the two spheres is denoted L. The dimensionless distance is defined by: $\xi = L/d$.

In the first case, the two spheres are in one another's wake. The flow is parallel to their centre lines. This arrangement is referred to as "spheres in the wake", and has the index 'w'. In the second case, flow is perpendicular to the centre lines. This arrangement is referred to as "side by side spheres" and has the index 's'.

The viscoplastic behaviour of the fluid used in this experiment may be represented by the Herschel–Bulkley viscoplastic model:

$$\begin{cases} \underline{\underline{\tau}} = 2\left(K\sqrt{-4D_{II}}^{\,n-1} + \dfrac{\tau_0}{\sqrt{-4D_{II}}}\right)\underline{\underline{D}}, & \text{if } \sqrt{-\tau_{II}} > \tau_0 \\ \underline{\underline{D}} = 0, & \text{if } \sqrt{-\tau_{II}} \le \tau_0 \end{cases} \tag{1}$$

with τ_0 being the yield stress, K the consistency, n the flow index, $\underline{\underline{D}}$ the rate of deformation tensor, D_{II} the second invariant of the rate of deformation tensor, $\underline{\underline{\tau}}$ the stress tensor deviator and τ_{II} is the second invariant of the stress tensor deviator.

However, it should be noted that the model fluid used in this study has an elasto-viscoplastic behaviour (see Section 4).

2.1. Isolated sphere

Let us first consider the case of an isolated sphere, to be used as the reference problem. An isolated sphere moving in a viscoplastic fluid at very low velocity creates stresses in its immediate vicinity of the order of the yield stress τ_0. The plastic drag coefficient of a sphere may therefore be expressed as:

$$Cd^* = \frac{F_d}{A \cdot \tau_0} \tag{2}$$

$A = \pi d^2/4$ represents the frontal area and F_d is the drag force of the sphere.

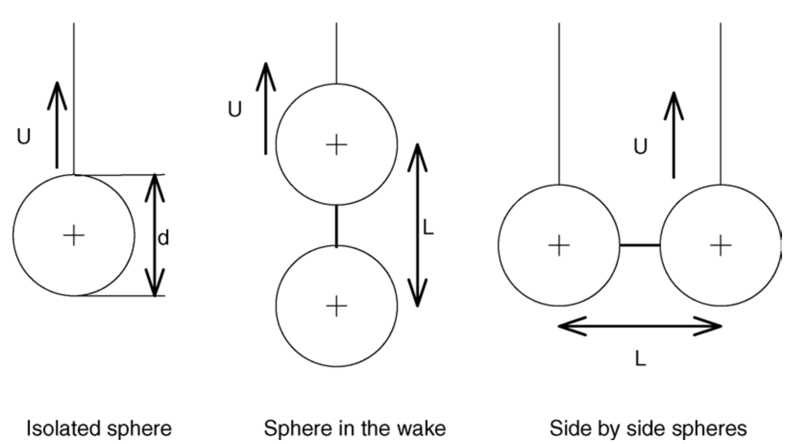

Fig. 1. Spatial position of spheres.