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Interfacial instability between sheared elastic liquids in a channel

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Abstract

We consider the linear stability of the interface between two sheared elastic liquids at large Weissenberg number (*Wi*) with negligible inertia. The liquids are of Oldroyd-B or UCM type and have matched viscosity. In UCM liquids, Renardy [Y. Renardy, Stability of the interface in two-layer Couette flow of upper convected Maxwell liquids, J. Non-Newton. Fluid Mech. 28 (1988) 99–115] found a purely elastic instability for short-waves in the absence of surface tension for which the perturbation flow decays exponentially away from the interface. For UCM liquids at large *Wi* we show that this instability persists even though the wavelength is larger than the channel width and the disturbance occupies the entire channel. Surprisingly, the growth rate is not affected by the location of the walls, even though the mode structure is altered. This analysis suggests a reappraisal of the appropriateness of the *short-wave* and *long-wave* classifications for instabilities of viscoelastic liquids in order to accommodate the additional length scale introduced by fluid velocity and relaxation. The instability persists for Oldroyd-B liquids even as the elastic contribution to viscosity approaches zero. Surprisingly too, the inclusion of surface tension does not affect the asymptotic growth rate at large wavenumber. When more modest values of *Wi* are considered, we find parameter values for which arbitrarily large surface tension reduces the growth rate but does not stabilize the flow; previously proposed mechanisms based on the interface displacement are therefore inadequate to explain the instability. Because the instability is locally generated, it appears in other high *Wi* flows with interfaces, both in channels and in pipes.

Keywords: Elastic instability; Interfacial instability; Channel flow; Surface tension

1. Introduction

Viscoelastic flows are important in a number of industrial applications and their instabilities have received considerable attention. The elasticity provides a source of energy for instabilities even in the absence of inertia, creating a class of *purely elastic* instabilities. Reviews of purely elastic instabilities can be found in [2,3]. In this paper, we study the stability to disturbances with wavenumber k of two inertialess Upper Convected Maxwell (UCM) or Oldroyd-B liquids. The liquids undergo shear in a channel of width L with characteristic velocity U_0; their viscosities are matched, but their relaxation times differ.

Much of the theoretical investigation of inertialess interfacial instabilities in viscoelastic liquids began with Chen [4] in the long-wave (wavelength long compared to channel width: $L \ll k^{-1}$) limit and with Chen & Joseph and Renardy [1,5] in the short-wave (wavelength short compared to channel width: $k^{-1} \ll L$) limit. The physical mechanism behind the long-wave instability was provided by Hinch et al. [6]. Related theoretical work in both limits was done by Ganpule and Khomami [7–9]. The results were generalized for other liquids by Wilson [10] and Wilson and Rallison [11–13].

Inertialess Couette flow of two Newtonian liquids with matched viscosity is linearly stable, as is the inertialess Couette flow of a single Oldroyd-B liquid [14,15]. A nonlinear stability proof for UCM liquids in Poiseuille flow is claimed by [16] who showed that the flow minimized an energy functional, but recent work [17,18] shows that no reasonable energy functional will decay monotonically in time for Oldroyd-B or UCM liquids. This is further confirmed by some numerical simulations [19] which find a finite amplitude nonlinear instability of Poiseuille and Couette channel flow for Oldroyd-B liquids when the elastic component of viscosity is large compared to the Newtonian component of viscosity.

Because the Couette flow of a single inertialess Oldroyd-B liquid is linearly stable, the short and long-wave interfacial instabilities must be attributed to the jump in elastic properties at the

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interface. The wavespeed of the long-wave mode relative to the interface is found to be much less than the velocity scale defined by the wavelength and the growth rate, that is, at leading order the wave remains stationary relative to the interface. This reflects the fact that the instability can be explained in terms of the normal stress jump which is independent of the sign of the shear rate [6]. In contrast, the short-wave mode travels with a relative speed comparable to the velocity scale defined by its growth rate and wavelength. The physical mechanism must involve some effect which depends on the sign of the shear rate. Some mechanisms have been suggested for this instability that depend on interface displacement [9,20]. We show, however, that at sufficiently large Weissenberg number this instability exists even if the interface is held flat by surface tension, so a different explanation is needed.

Renardy [1] considered interfacial instabilities of inertialess Couette flow of UCM liquids for short-waves: $k^{-1} \ll L$. She found that the perturbed flow is localized in a boundary layer of thickness $1/k$ near the interface. Consequently short-wave instabilities exist provided the walls are sufficiently far from the interface. In the large Weissenberg number limit, she found that the growth rate is a function only of the ratio of the two relaxation times. For some ratios the flow is stable. In contrast, at low Weissenberg number, all pairs of relaxation times are unstable. The two limits involve different mechanisms. This paper focuses on the large Weissenberg number limit.

Chen and Joseph [5] examined inertialess core-annular flow of UCM liquids through a pipe without surface tension. They found the same short-wave behavior as Renardy because the curvature of the pipe disappears from the asymptotic equations. With surface tension they claim that the flow stabilizes at large enough k. Our results disagree with this conclusion.

Wilson and Rallison [11] generalized the UCM results to Oldroyd-B liquids, again with $k^{-1} \ll L$. They found that the addition of a Newtonian component to the viscosity has a destabilizing effect. In the limit where the Newtonian viscosity is large compared to the elastic stress, they found instability whenever the relaxation times of the two liquids are different. In the presence of surface tension at large enough k they showed that the normal force due to surface tension dominates the elastic normal force which suggests that the interface and hence the flow should be stabilized. However, we show that at large *Wi* the normal force balance is irrelevant to the stability.

We consider Couette flow through a channel of width L with walls moving at a relative velocity of U_0. In characterizing the different classes of interfacial instability it is important to recognize that for viscoelastic liquids in Couette flow three length scales enter the problem: the channel width L, the wavelength of the disturbance k^{-1}, and the relative distance $U_0\tau$ travelled by the walls in a relaxation time τ. This final length scale is a measure of the distance a typical particle travels during a relaxation time. Other length scales can be constructed from these three. For example liquid particles initially separated by the distance $2\pi L/U_0\tau k$ in the cross-stream (y) direction are separated by a wavelength in the streamwise (x) direction after a relaxation time. We find later that the length scale $L/U_0\tau k$ determines the thickness of boundary layers in the flow.

The previous analyses considered $k^{-1} \ll L$ (short-waves) or $k^{-1} \gg L$ (long-waves) and implicitly assumed that $k^{-1} \ll U_0\tau$ for short-waves or $k^{-1} \gg U_0\tau$ for long-waves. This leaves two other limits unexplored: $L \ll k^{-1} \ll U_0\tau$ and $U_0\tau \ll k^{-1} \ll L$. In the latter case the Weissenberg number $Wi = U_0\tau/L$ is small and the elastic effects are weak; the analysis of [1,5] for $k^{-1} \ll U_0\tau \ll L$ applies to this case. This paper focuses on the unexplored former case for which $Wi \gg 1$. In this regime the wavelength is long compared to the channel width, but short compared to the relaxation length scale. This leads to a mixture of short and long-wave properties, allowing us to use standard short-wave techniques, but also to make standard long-wave assumptions (e.g., the pressure gradient varies only in the x-direction).

The organization of this paper is as follows: In Section 2 we describe the governing equations and the unperturbed Couette flow. In Section 3 we study the large *Wi* limit of the UCM liquid analytically and numerically, and in Section 4 we study the large *Wi* limit of the Oldroyd-B liquid numerically. We then discuss the effect of surface tension, showing in Section 5 that even for moderate *Wi* some flows are not stabilized by arbitrarily large surface tension. In Section 6 we discuss the physical scalings of the instability. An additional instability is briefly analyzed in Section 7. In Section 8 we show that the main instability of this paper is robust in that it persists for other flow profiles under some mild assumptions. Finally, in Section 9 we offer some concluding remarks.

2. Governing equations

Consider two incompressible Oldroyd-B liquids in steady Couette flow in a channel of width L as shown in Fig. 1. We choose the origin in y to be the location of the unperturbed interface. The frame of reference is chosen to travel with the interface velocity. The lower liquid occupies a fraction Δ of the channel; the walls at $y = (1-\Delta)L$ and $y = -\Delta L$ move horizontally with velocity $(1-\Delta)U_0$ and $-\Delta U_0$ respectively.

The liquids have different relaxation times τ_- and τ_+ but the same, constant, shear viscosity μ, as well as the same relative contributions of elastic and Newtonian components to that viscosity. Without loss of generality we take $\tau_- \geq \tau_+$. In the absence of inertia we have

$$\nabla \cdot \Sigma = \mathbf{0}, \quad (1)$$

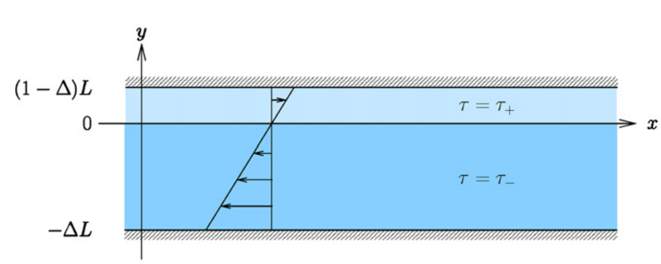

Fig. 1. Two elastic liquids in Couette flow $U = U_0 y/L$ through a channel. The liquids differ only in relaxation time τ.