J. Non-Newtonian Fluid Mech. 143 (2007) 107–119

Journal of Non-Newtonian Fluid Mechanics

www.elsevier.com/locate/jnnfm

Bread dough rheology and recoil 2. Recoil and relaxation

Roger I. Tanner *, Shao-Cong Dai, Fuzhong Qi

School of Aerospace, Mechanical and Mechatronic Engineering, University of Sydney, Sydney, NSW 2006, Australia

Received 14 November 2006; received in revised form 1 February 2007; accepted 2 February 2007

Abstract

It is shown that a simple Lodge model with a power-law memory function and a damage function which is a function of strain can describe the behaviour of a bread dough in steady shear, steady elongation, small and medium size sinusoidal strains, and shear stress relaxation. The number of parameters needed to be found from experiment is minimal, which is a clear advantage. Some questions remain about the frequency dependence in the sinusoidal oscillatory test. We also use the model to describe the difficult, but practically important problem of recoil after steady elongation. By a simple modification of the damage function concept for the recoil phase, we are able to describe the results of experiments at elongation rates between 0.001 and 0.1 s^{-1} and Hencky strains up to 2.5. Finally, although no explicit yield stress has been introduced into the model, results resemble those from models with a yield stress that depends on the rate of elongation.

Keywords: Bread dough; Viscoelasticity; Recoil; Relaxation; Damage function

1. Modelling of bread dough

The present paper continues the exploration begun in Part 1 [1] of the use of a modified Lodge elastic fluid model with a damage function to describe the mechanics and rheology of bread dough. In Part 1 [1] we showed that the rheology of this soft, starch-filled solid could be described in suddenly started steady shear and elongation by a constitutive equation of the Lodge [2] form:

$$\boldsymbol{\sigma} + P\mathbf{I} = f \int_{-\infty}^{t} m(t-t')\mathbf{C}^{-1}(t')\,\mathrm{d}t' \qquad (1)$$

where $\boldsymbol{\sigma}(t)$ is the stress at time t, P the pressure, $\mathbf{I}$ the unit tensor and $\mathbf{C}^{-1}(t')$ is the Finger strain tensor [2,5] at time t' computed relative to the configuration at the present time t. The memory function is assumed to be of the power-law form:

$$m(t) = pG(1)t^{-(1+p)} \qquad (2)$$

where the constants p and $G(1)$ can be found from small-strain oscillatory testing [1]. (Note that the constant $G(1)$ here signifies the numerical value of the stress relaxation modulus $G(t)$ at $t = 1$ s—the constant does not have the dimensions of stress.) The "damage function" f is assumed to be a function of the Hencky strain at time t, computed relative to the state of rest, which is assumed to occur for times less than zero.

For the JANZ dough used for the tests, we have already [1] reported on preparation methods, sinusoidal small-strain, shear and elongation beginning from rest, and on sinusoidal strain tests of moderate magnitude (∼10% strain). Here we present, for the same dough, relaxation after a shear strain applied at $t = 0$, and recoil in elongation from various initial strains. These experiments form, we believe, a unique set for a single dough, and we attempt to use Eqs. (1) and (2) to describe all the data. Two characteristics of the model are the economy of parameters, and the rapid decrease of the damage function f from unity at very small strains (∼0.001 or smaller) to a value of order 0.1 at strains of only 10–20% [1]. This remarkable softening with work input (kneading) is of course a well-known and valuable aspect of dough rheology. We begin by studying relaxation from a known shear strain, then consider the larger-strain oscillatory behaviour reported in Part 1 [1], and finally discuss recoil after elongational stress and release.

2. Stress relaxation

Ideally, in this test a suddenly applied (step) of shear strain (γ_0) is applied at $t = 0$, and the decay of shear stress is measured.

* Corresponding author. Tel.: +61 2 9351 7153; fax: +61 2 9351 7060.
E-mail address: rit@aeromech.usyd.edu.au (R.I. Tanner).

0377-0257/$ – see front matter
doi:10.1016/j.jnnfm.2007.02.001

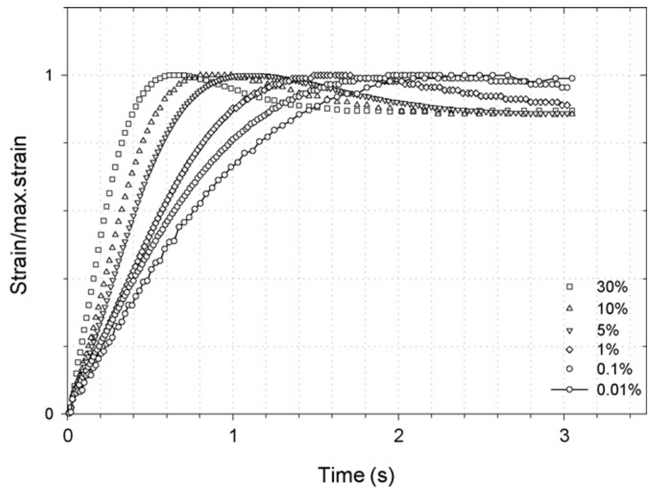

Fig. 1. The shear strain–time curves achieved for various sized "steps" of shear. The maximum shear is used to normalize the curves. The ideal test would apply a true step at $t=0$. The sizes of the steady-state shear steps (0.0001–0.3) are noted in the figure.

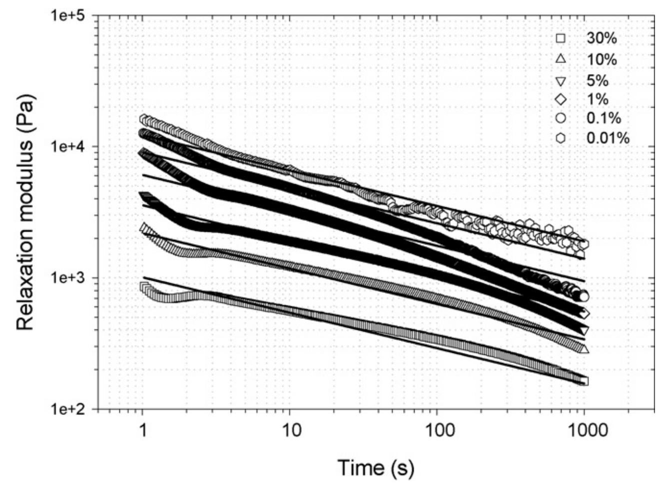

Fig. 2. Stress relaxation modulus after initial shear deformation. The final strain (γ_0) is only reached after $t \simeq 1$ s The decay curves are fairly well described by power-law curves of the form $f(\gamma_0)G(1)t^{-p}$ for $t \gg 1$ s. The ordinate is τ/γ_0, the apparent shear relaxation function. The fitted lines all have the same slope $t^{-0.27}$.

In shear, Eqs. (1) and (2) can be written in the form:

$$\tau = f \int_{-\infty}^{t} G(1)(t-t')^{-p}\dot{\gamma}(t')\,\mathrm{d}t' \tag{3}$$

and the result for a step of shear of size (γ_0) is

$$\tau = \gamma_0 f(\gamma_0) G(1) t^{-p} \tag{4}$$

In experiments a true step of shear is not available, and Fig. 1 shows that around 1–2 s is needed to reach the final strain γ_0 in the Paar Physica MCR 300 instrument. There is also a variation of strain (and stress) with radius in the parallel plate apparatus, so here γ_0 and τ are actually taken as the strain and stress at 75% of the plate radius. It is known that this approximation is usually very little ($O(1\%)$) in error [3]. If we idealize the response of Fig. 1 as a constant ramp $\dot{\gamma} = \gamma_0/t_0$, where t_0 is the time to reach steady state and $\gamma = \gamma_0$ for $t > t_0$, we find the response at t ($>t_0$) to be

$$\tau = \frac{\gamma_0 f(\gamma_0) G(1)}{t_0(1-p)}[t^{1-p} - (t-t_0)^{1-p}] \tag{5}$$

and for $t \gg t_0$ we find:

$$\tau = \gamma_0 f(\gamma_0) G(1) t^{-p}\left[1 + \frac{p}{2}\left(\frac{t_0}{t}\right) + O\left(\frac{t_0}{t}\right)^2\right] \tag{6}$$

Since in our case $p=0.27$, the result (6) is within 1% of (4) for $t/t_0 \geq 10$.

In fact the power-law asymptote will always be reached for large enough times. The present set of experiments shows behaviour close to the power-law relaxation at quite early times (Fig. 2), and in this figure we have fitted the curves shown of slope -0.27 to the stress data at $t=10$ s. This enables us to find $f(\gamma_0)$, since we assume $G(1)=12.2\,\mathrm{kPa\,s}^{0.27}$, from the step strain of 0.01% (top curve in Fig. 2). (Previously we found $G(1) \sim 10.7\,\mathrm{kPa\,s}^{0.27}$ from oscillation; so there is reasonable concordance in these values.) Recalling that the Hencky strain $\boldsymbol{\varepsilon}_{\mathrm{H}} \sim (1/2)\gamma$ for small strains [4], we can plot $f(\varepsilon_{\mathrm{H}})$ from our relaxation tests (circles in Fig. 3). From Fig. 7 of Part 1 [1] we find the points shown as inverted triangles (steady shear) and squares (steady elongation).

It is noticeable that for very long times ($\geq$100 s) there is some softening present that is not predicted by the model. Looking at the small-strain oscillation results of Part 1 (Fig. 2) we see that these tests also show some deviation from a strict power-law at very low frequencies (<0.1 rad/s), and since measurements were not made below 0.06 rad/s, there may be effects at these low frequencies and long times which are not modelled correctly. However, for many process applications these long times/low frequencies will often be unimportant. We can also interpret these results using a KBKZ model. From well-known results [5] if the strain–time separation of the KBKZ kernel is assumed, then the

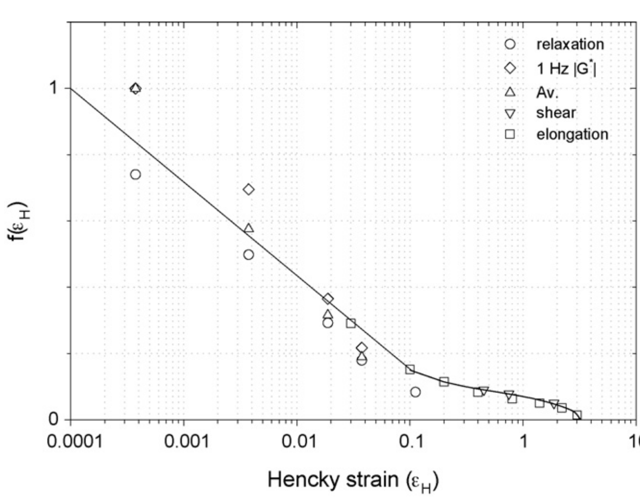

Fig. 3. Damage function f as a function of the Hencky strain ε_{H} for stress relaxation measurements (◯), finite-amplitude strain oscillations at 1 Hz (◇) and the average of 0.01, 1 and 30 Hz measurements (△), steady shearing (▽) and steady elongation results (□) from Part 1 [1] are also shown. For small ε_{H} (<0.05) the full line shows $f=-(0.135+0.123\ln\varepsilon_{\mathrm{H}})$; for larger ε_{H} values, the values for shear and elongation are shown in Fig. 7, Part 1 [1]; the formula for f in this range is given in Appendix A (Eq. (A4)).